Learning with Large-Scale Social Media Networks

by

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ABSTRACT

Social media such as blogs, Facebook, Twitter, YouTube and Flickr enables people of all walks of life to express their thoughts, voice their opinions, and connect to each other more conveniently than ever. The boom of social media opens up a vast range of possibilities to study human interactions and collective behavior on an unprecedented scale. This dissertation presents a framework for learning with large-scale social media networks in order to understand human interactions and to predict collective behavior. Network interactions are typically heterogeneous, representing disparate relations, but most social media sites present only connections with no or limited relation information. Hence, social dimension is introduced to differentiate heterogeneous relations. A learning approach based on social dimensions is proposed, achieving substantial improvement over the state of the art. It is then extended to unify some unsupervised learning methods to handle networks with various types of entities and interactions. As social media networks are often of colossal size, an edge-clustering method is proposed to extract sparse social dimensions in order to address the scalability challenge. In sum, this research provides novel concepts and efficient algorithms to harness the power of social media networks, enables the integration of data in heterogeneous format and information from networks of multiple modes or dimensions, and offers a learning-based solution to social computing.

To My Parents and Xinru

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INTRODUCTION

The past decade has witnessed a rapid development and change of the Web and Internet. The advancement in computing and communication technologies is drawing people together in innovative ways. Numerous participatory web and social networking sites have been cropping up, empowering new forms of collaboration, communication and emergent intelligence. Prodigious numbers of online volunteers collaboratively write encyclopedia articles of previously impossible scopes and scales; Online marketplaces recommend products by investigating user shopping behavior and interactions; Political movements are also creating new forms of engagement and collective action. The boom of social media opens up a vast range of possibilities to study human interactions and collective behavior on an unprecedented scale. This chapter first introduces social media and its characteristics, then defines the task of leveraging social media networks for learning, followed by a discussion of challenges associated with the task.

1.1 Social Media

With the pervasive availability of Web 2.0 and social networking sites, people can interact with each other easily through various social media. Table 1.1 lists assorted forms of social media sites, including wikis, social networking sites, social bookmarking sites, media sharing sites, social news, Blogs, microblogging platforms, and forums. Though they may look quite different, they share one common feature that distinguishes them from the classical web and traditional media: the "consumers" of content or information online are also the "producers". Essentially, everybody in social media can be an information outlet [114], resulting in mountains of user-generated content.

This new type of mass publication enables the production of timely and grassroots information. This was evidenced in the London terrorist attack in 2005, during which some witnesses blogged their experience to provide first-hand reports of the event [137]. Another example was the bloody clash ensuing the Iranian presidential election in 2009, for which many provided live updates on Twitter, a microblogging platform. Social media also allows collaborative writing to produce high-quality bodies of work otherwise impossible. Take Wikipedia as an example. "Since its creation

Table 1.1: Various Forms of Social Media

Wikis	Wikipedia, Scholarpedia, ganfyd, AskDrWiki
Social Networking	Facebook, MySpace, LinkedIn, Orkut, PatientsLikeMe
Social Tagging	Del.icio.us, StumbleUpon
Media Sharing	Flickr, YouTube, Justin.tv, Ustream, Scribd
Social News	Digg, Reddit
Blogs	Wordpress, Blogspot, LiveJournal, BlogCatalog
Microblogging	Twitter, foursquare
Forums	Yahoo! answers, Epinions

in 2001, Wikipedia has grown rapidly into one of the largest reference web sites, attracting around 65 million visitors monthly as of 2009. There are more than 85,000 active contributors working on more than 14,000,000 articles in more than 260 languages[1]."

Another distinctive characteristic of social media is its rich user interaction. The success of social media relies on the engagement of users. More interactions encourage more user participation, and vice versa. For example, Facebook claims to have more than 500 million active users as of July 20, 2010[2]. The huge amount of user participation and interaction result in eight social media sites in the top 20 websites as shown in Table 1.2. This rich user interaction forms a connected *network* of users, providing unparalleled opportunities to study human interaction and collective behavior. Many computational challenges ensue, urging the development of advanced computational techniques and algorithms.

Table 1.2: Top Websites in US based on Internet Traffic as reported by Alexa on 7/10/2010

Rank	Site	Rank	Site
1	google.com	11	**blogger.com**
2	**facebook.com**	12	msn.com
3	yahoo.com	13	go.com
4	**youtube.com**	14	**myspace.com**
5	amazon.com	15	aol.com
6	**wikipedia.org**	16	bing.com
7	craigslist.org	17	espn.go.com
8	ebay.com	18	**linkedin.com**
9	**twitter.com**	19	cnn.com
10	live.com	20	**wordpress.com**

[1]http://en.wikipedia.org/wiki/Wikipedia:About
[2]http://www.facebook.com/press/info.php?statistics

1.2 Learning with Social Media Networks

Millions of users in social media are playing, working, and socializing online. This offers vast troves of digital information for mining patterns about users, their friends, likes, dislikes, etc. In this dissertation, I investigate how we can leverage user interaction information to understand human interactions and predict collective behavior in social media. A clear understanding of social connections and collective behavior in social media has a broad range of applications. Take social networking advertising as an example. A common approach to targeted marketing is to build a model mapping from user profiles (e.g., the geography location, education level, gender) to ads categories. Since social media often comes with a friendship network between users and many daily interactions, we may be able to exploit this interaction information to infer more accurately about the ads that might attract a user. In order to understand how users interact with and influence each other, we address both unsupervised and supervised learning with social media networks.

1.2.1 Unsupervised Learning and Community Detection

Understanding human interaction is one of the core problems in conventional social network analysis [53, 147]. It aims to answer questions such as which groups of people tend to interact with each other more frequently? Why are two users connected? What propels a user to join a certain group? Given one group, what are the group norm [67] and shared profiles [126]? In the context of social media, we study a fundamental task: *how to extract communities (a.k.a., cohesive groups) from social media networks?*

This problem is known as community detection [43], or clustering on graphs [52]. This clustering problem is one form of *unsupervised learning*. But here data instances are presented in a network rather than conventional attribute format. Clustering on graphs has been well studied. Many algorithms have been proposed. Please see [123] and [43] for detailed surveys. For convenience, I also review several representative approaches and summarize them in a unified process in Appendix A.

Social media, however, presents characteristics that were seldom considered in earlier work. Various types of networks commonly coexist in social media. Interactions in social media are often multi-dimensional, multi-modal, and highly dynamic. One network may involve heterogeneous

types of entities. YouTube, for instance, includes users, videos, tags and comments. Users are also encouraged to interact with each other in various forms such as sending a message, leaving a comment, thumbing up or down for one's post, etc. Moreover, interactions between users can shift rapidly due to external events. Some communities in social media are highly dynamic, forming and dissolving swiftly. In sum, social media networks are often more than just a static friendship network. Correspondingly, it poses new challenges to extract communities by integrating all kinds of network information.

1.2.2 Supervised Learning

In a connected environment, individual behaviors tend to be correlated with his/her friends. For example, if our friends buy something, there is a better-than-average chance that well buy it, too. Such correlation between user behavior can be attributed to both *influence* and *homophily* [92]. When an individual is exposed to a social networking environment, their behavior or decision are likely to be influenced by each other. This influence naturally leads to correlations between connected users. On the other hand, we tend to link up with one another in ways that confirm rather than test our core beliefs. In other words, we are more likely to connect to others sharing certain similarities. Consequently, it is no wonder that connected users demonstrate correlations in terms of behavior.

Since a social network provides valuable information concerning actor classes (one class refers to users of the same behavior, preference or property), it is natural to ask how we can use the correlation presented in a social network to predict classes of users. That is, given a social network with class information of some actors, how can we infer the class of the remaining actors within the same network? This supervised learning problem assumes that we can observe classes of some individuals so that social learning is attainable. The amount of information that we can collect depends on tasks. For instance, if we want to know whether a user will click on an ad, we can collect this information when the ad is displayed to the user. To determine behavior concerning voting for a presidential candidate, we can collect some voluntary responses using online surveys. With such class information, we might be able to unravel the class of other users with no class information.

Both supervised and unsupervised learning tasks are complementary to each other. As we show later, unsupervised learning is one key step in our proposed supervised learning approach. On the

other hand, the success of the proposed supervised learning framework sheds lights for us to unify unsupervised learning for various kinds of social media networks.

1.3 Challenges

The analysis of social structures based on network topology is not new. That is exactly what conventional social science [147] trying to address. As social media thrives, many challenges arise. They were seldom encountered and addressed in traditional social sciences, thus novel approaches have to be developed.

A traditional social science study often involves the circulation of questionnaires, asking respondents to detail their interaction with others. Then a network can be constructed based on the response, with nodes representing individuals and edges the interactions between them. This type of data collection confines most traditional social network analysis to a limited scale, typically hundreds of actors at most in one study. Various relations can present in one network, and relations between actors are typically explicitly known, e.g., actor a is the mother of actor b; actors b and c are colleagues.

Social media, on the one hand, provides readily-available interactions between human beings on an unprecedented scale. For example, Leskovec and Horvitz [77] analyzed a network constructed based on instant messenger communication between 180 million users. This kind of scale can hardly be possible in traditional social sciences, thus offering a new opportunity to study human interactions and behavior in the large. On the other hand, social media also poses some challenges to be addressed:

- Limited information. Diverse relations are intertwined with connections in a social network. One user, for instance, might connect to her friends, relatives, college classmates, colleagues, or online buddies with similar hobbies. Connections in a social network can represent various kinds of relationship between users. However, when a network is collected from social media, most of the time, no explicit information is available why these users connect to each other and what their relationships are. This missing relation information can limit the performance of some techniques when they are applied to a social media network.

- Scalability. Networks in social media can be huge, often involving millions of actors and

hundreds of millions of connections, while traditional network analysis normally deals with hundreds of subjects or fewer. Twitter, for example, claims to have more than 100 million users, and Facebook 500 million active users. Existing network analysis techniques might fail to handle networks of this astronomical size.

- Heterogeneity. In social media, it is very likely that heterogeneous types of interaction exist between the same set of users. Moreover, multiple types of entities can also be involved in a network. Take YouTube as an example, users, tags, videos are all weaved into the same network. Analysis of social media networks with heterogeneous entities and interactions requires new theories and tools.

- Evolution. Social media emphasizes timeliness. For example, in content sharing sites and blogosphere, people quickly lose their interest in most shared contents and blog posts. This differs from classical web mining. It is common that new users jump in, new connections establish between existing members, and old users become dormant or simply leave. Behind noisy interactions, communities can also emerge, grow, shrink, or dissolve. How can we capture the dynamics of individuals and communities? Can we find the die-hard members that are the backbone of communities and determine the rise and fall of their communities?

- Evaluation. A research barrier concerning mining social media is evaluation. In traditional data mining, we are so used to the training-testing model of evaluation. It differs in social media. Since many social media sites are required to protect user privacy information, limited benchmark data is available. Another frequently encountered problem is lack of ground truth for many social computing tasks, which further hinders some comparative study of different works. Without ground truth, how can we conduct fair comparison and evaluation?

1.4 Roadmap

Before I discuss a framework for learning with large-scale social media networks, I present a cursory overview of the dissertation.

In chapter 2, we formulate the problem of supervised learning with social media networks, and review existing state-of-the-art algorithms. We pinpoint limitations of these methods when they are

Table 1.3: Challenges Discussed in Each Chapter

Challenges	Chapter 2 Supervised Learning	Chapter 3 Unsupervised Learning	Chapter 4 Scalable Learning
Limited Information	✓		✓
Scalability			✓
Heterogeneity		✓	
Evolution		✓	✓
Evaluation		✓	

applied to social media networks. Then, we propose a social dimension based learning framework for classification with network data and present comprehensive empirical results.

Based on the concept of social dimension and the core idea in the proposed learning framework, we show that the framework can be extended to handle unsupervised learning with various types of social media networks in Chapter 3. We start from one type of network with heterogeneous interactions, and derive a unsupervised learning approach following the proposed social dimension concept in the previous chapter. This approach is then generalized to handle networks of heterogeneous entities, interactions or evolutions. We also demonstrate that such a simple unsupervised learning approach corresponds to a sensible objective function through a series of derivation. This deepens our understanding of the framework. Due to the lack of ground truth, novel evaluation strategies are also presented.

Since most social media networks are often huge, it is imperative to develop scalable methods. Chapter 4 discusses the possibilities of extracting sparse social dimensions in order to achieve scalable learning of collective behavior. We propose a simple yet effective algorithm. It is able to extract sparse social dimensions from a network of millions of nodes in minutes, significantly advancing the applicability of our proposed framework in dealing with large-scale networks.

Table 1.3 summarizes the challenges we are trying to address in these chapters. In essence, this research provides the concept of social dimension, as well as efficient algorithms to harness the predictive power of social media networks. It enables the integration of data in heterogeneous format and information from networks of multiple modes or dimensions, and offers a novel and feasible solution for social computing. In Chapter 5, we conclude and point out some future directions based on findings and lessons learned from this research.

Chapter 2

SUPERVISED LEARNING WITH SOCIAL MEDIA NETWORKS

With the rapid development of participatory web and social networking sites like YouTube, Twitter, and Facebook, social media has reshaped the way people interact with each other. This blossom of social networks provides opportunities to predict user-related attributes or behavior. In this chapter, we investigate the task of supervised learning with social media networks. In particular, how we can leverage social media networks to infer user attributes or behavior in the network.

It is noticed that diverse relations are intertwined with connections in social media networks. For example, one user might connect to her relatives, college classmates, colleagues, or online buddies with similar hobbies. The connections in a social network are inherently *heterogeneous*, representing various kinds of relationship between users. However, when a social network is collected from social media, most of the time, no explicit information is available why these users connect to each other and what their relationships are. Existing methods that address classification problems with network data seldom consider this heterogeneity. Direct application of existing methods treats connections homogeneously, though they are inhomogeneous. This can lead to an unsatisfactory performance Hence, we propose to differentiate connections and employ different relations in building a discriminative classifier for classification.

We present *social dimension*, a concept dealing with connection heterogeneity, and propose a classification framework (denoted as *SocioDim*) based on latent social dimensions. Each dimension can be considered as the description of potential affiliations of social actors, which accounts for the their interactions. With these social dimensions, we can take advantage of the power of discriminative learning such as support vector machines or logistic regression to automatically select relevant social dimensions for classification. The proposed learning framework is flexible to allow the plug-in of different modules. It is demonstrated that our framework outperforms alternative representative methods on social media data. SocioDim also offers a simple yet effective approach to integrating network information with other features associated with actors such as social content or profile information. Such a learning framework also applies to other domains with network data.

2.1 Problem Statement

In this chapter, we study supervised learning with networked data. For instance, in an advertising campaign, advertisers attempt to deliver ads to those users who are interested in their products, or similar categories. The outcome of user interests can be represented using + or -, with + denoting a user is interested in the product and - otherwise. We assume that the interests of some users are already known. This can be extracted from user profiles or their response to a displayed ad. The task is to infer the preference of the remaining users within the same network.

Individual interests cannot be captured by merely one class. It is normal to have multiple interests in a user profile. Rather than concentrating on univariate cases of classification in networked data [90] (each node has only one class label), here we examine a more challenging task that each node in a network can have multiple labels, i.e., a multi-label classification problem [142] or a multi-task learning problem [21]. In this setup, the univariate classification is just a special case. Following other standard setup for collective classification [112], the multi-label classification problem with network data can be formally described below:

Given:

- K categories $\mathscr{Y} = \{\mathscr{Y}_1, \cdots, \mathscr{Y}_K\}$;
- a network $\mathscr{A} = (V, E, Y)$ representing the interactions between nodes, where V is the vertex set, E is the edge set, and each node v_i is associated with class labels $\mathbf{y}_i$ whose value can be unknown;
- the known labels Y^L for a subset of nodes V^L in the network, where $V^L \subseteq V$ and $y_{ij} \in \{+, -\}$ denotes the class label of the vertex v_i with respect to category $\mathscr{Y}_j$.

Find:

- the unknown labels Y^U for the remaining vertices $V^U = V - V^L$.

Each vertex in the network represents one actor or one user in social media. Thereafter, data instances, actors, vertices, nodes, entities and objects are used interchangeably in the context of a network.

The problem above is referred as within-network classification [90]. The classification is based on network information alone. In reality, there might be features associated with each node (actor). For example, in blogosphere, the content of blog posts are available besides the blog network. While it is an essential task to piece together these distinctive types of information, it is not the main focus here. In this chapter, we examine different approaches to classification based on network information alone, and then discuss their extensions to include actor features for learning.

2.2 Motivation

We briefly review *collective inference*, a commonly used method that has been proposed to address classification with network data, and discuss its limitations when it is applied directly to networks in social media.

2.2.1 Collective Inference

When data instances are connected in a network, they are not identically independently distributed (i.i.d.) as in conventional data mining. It is empirically demonstrated that linked entities have a tendency to belong to the same class [90]. This correlation in the class variable of connected objects can be explained by the concept of *homophily* in social science [92]. Homophily suggests a connection between similar people occurs at a higher rate than among dissimilar ones. It is one of the first characteristics studied by early social network researchers [5, 148, 18], and holds for a wide variety of relationships [92]. Homophily is also observed in social media [40, 138, 76].

Based on the empirical observation that labels of neighboring entities (nodes) are correlated, the prediction of one node cannot be made independently, but also depends on its neighbors. To handle the interdependency, collective inference is widely used to address the classification problem in networked data [64, 90, 112]. A common Markov assumption is that, the labels of one node depends on the labels (plus other attributes if applicable) of its neighbors. In particular,

$$P(y_i|\mathscr{A}) = P(y_i|\mathscr{N}_i) \tag{2.1}$$

where $\mathscr{A}$ is the network, y_i the label of node v_i, and $\mathscr{N}_i$ a set of its "neighbors". The neighbors are typically defined as nodes that are 1-hop or 2-hop away from v_i in the network [64, 44]. For training, a relational classifier based upon the labels (plus other available node attributes) of neigh-

bors is learned via the labeled nodes V^L. For prediction, collective inference [64] is applied to find an equilibrium status such that the inconsistency between neighboring nodes in the network is minimized. Relaxation labeling [25], iterative classification [87] and Gibbs sampling [46] are the commonly used techniques. All the collective inference variants share the same basic principle: it initializes the labels of unlabeled nodes V^U, and then applies the constructed relational classifier to assign class labels (or update class membership) for each node while fixing the labels (or class membership) of its neighboring nodes. This process is repeated until convergence.

2.2.2 Heterogeneous Relations

A social network is often a composite of various relations. People communicate with their friends online. They may also communicate with their parents or random acquaintances. The diversity of connections indicates that two connected users do not necessarily share certain class labels. When relation type information is not available, directly applying collective inference to such a network cannot differentiate connections between nodes, thus fails to predict the class membership of actors in the network. Let us look at a concrete example in Figure 4.1. Actor 1 connects to Actor 2 because they work in the same IT company, and connects to Actor 3 because they often meet each other in the same sports club. Given the label information that Actor 1 is interested in both Biking and IT Gadgets, can we infer Actors 2 and 3's labels? Treating these two connections homogeneously, we guess that both Actors 2 and 3 are also interested in biking and IT gadgets. But if we know how Actor 1 connects to other actors, it is more reasonable to conjecture that Actor 2 is more interested in IT gadgets and Actor 3 likes biking.

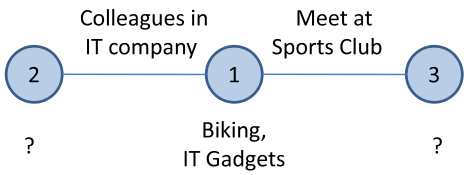


Figure 2.1: A Toy Example

The example above assumes the cause of connections is explicitly known. But this kind of information is rarely explicit in real-world applications, though some social networking sites like Facebook do ask connected users about the reason why they know each other. Most of the time,

only network connections (as in Figure 2.2) are available. If we can somehow differentiate the connections into different affiliations (as shown in Figure 2.3) and find out which affiliation is correlated more with the targeted class label, we can infer the class membership of each actor more precisely. Notice that an actor can present in multiple affiliations, e.g., actor 1 belongs to both Affiliation-1 and Affiliation-2 in the example.

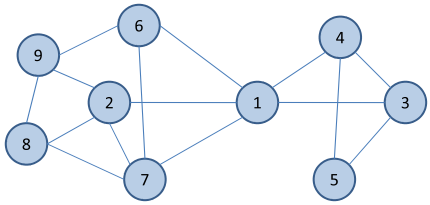


Figure 2.2: Node 1's local Network

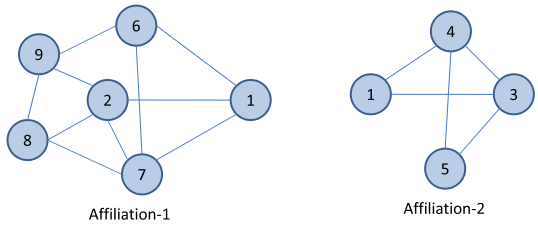


Figure 2.3: Different Affiliations

Given a network, differentiating its connections into distinct affiliations is not an easy task as the same actor is involved in multiple affiliations. Moreover, the same connection can be associated with more than one affiliation. For instance, one can connect to another as they are colleagues as well as going to the same sports club frequently. Instead of capturing affiliations among actors via differentiating connections directly, we resort to latent social dimensions, with each dimension representing a plausible affiliation of actors. Next, we introduce the concept of social dimensions, and illustrate a classification framework based on that.

2.3 SocioDim: A Learning Framework based on Social Dimensions

To handle network heterogeneity as we have mentioned in the previous section, we propose to extract *social dimensions* [121, 124] to capture the latent affiliations of actors and utilize them for classification. Below, we introduce the concept of social dimensions and a fundamental assumption for our framework.

Social dimensions are the vector-format representation of actors' involvement in different affiliations. Given the extracted affiliations in Figure 2.3, we can represent them as social dimensions in Table 2.1. If an actor is involved in one affiliation, then the entry of social dimensions corresponding to the actor and the affiliation is non-zero. Note that one actor can participate in multiple different affiliations (e.g., Actor 1 is associated with both affiliations). Different actors participate

in disparate affiliations in varying extent. So weighted values, instead of boolean values, can also be used to represent affiliation membership.

Table 2.1: Social Dimensions Corresponding to Affiliations in Figure 2.3

Actor	Affiliation-1	Affiliation-2
1	1	1
2	1	0
3	0	1
...	...	...

We assume one actor's label depends on its latent social dimensions. Specifically, we assume

$$P(y_i|\mathscr{A}) = P(y_i|S_i) \tag{2.2}$$

where $S_i \in \mathbb{R}^k$ denotes the social dimensions (latent affiliations) of node v_i. This is fundamentally different from the Markov assumption in Eq.(2.1) used in collective inference. Collective inference assumes the labels of one node relies on that of its neighbors. It does not capture the weak dependency between nodes that are not close or directly connected. Here we assume the labels are dependent on its latent social dimensions so the nodes within the same affiliations tend to have similar labels even though they are not directly connected. Based on the assumption in Eq. (2.2), we propose a learning framework SocioDim to handle the network heterogeneity for classification. The overview of the framework is shown in Figure 2.4. It is composed of two phases: we first extract the latent social dimensions S_i for each node, and then build a classifier based on the extracted dimensions to learn $P(y_i|S_i)$.

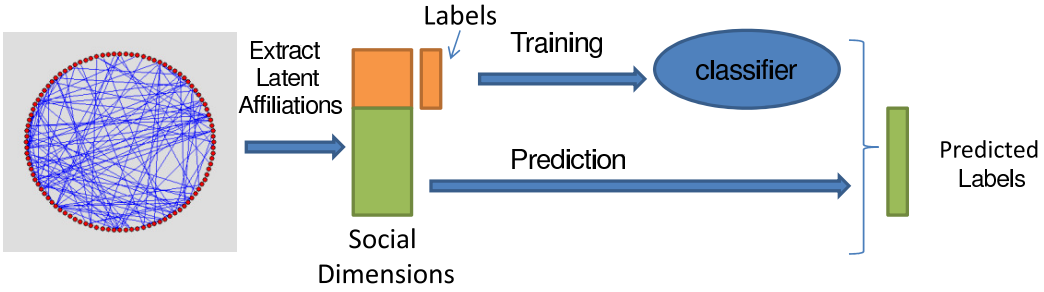
2.3.1 Phase I: Extraction of Social Dimensions

For the first phase, we require the following:

- an undirected network represented as a sparse matrix $A \in \mathbb{R}^{n\times n}$,
- the number of social dimensions to extract k.

The output should be social dimensions $S \in \mathbb{R}^{n\times k}$ of all nodes in the network. It is desirable that the extracted social dimensions satisfy the following properties:

- Informative. The social dimensions should be indicative of latent affiliations of actors.



Input: A social network $\mathscr{A}$,
the labels of some nodes in the network Y^L,
the number of social dimensions to extract k;
Output: the labels of unlabeled nodes Y^U.

1. given $\mathscr{A}$ and k, extract social dimensions $S \in R^{n \times k}$ via soft clustering;
2. based on S^L and Y^L, construct a discriminative classifier C;
3. based on S^U and C, output Y^U for unlabeled nodes.

Figure 2.4: SocioDim: A Classification Framework based on Social Dimensions

- Plural. The same actor can be involved in multiple affiliations, thus having non-zero entries in different social dimensions.
- Continuous. The actors might have different degree of associations to one affiliation. Hence, a continuous value rather than discrete $\{0, 1\}$ is more favorable.

One key observation is that when actors belong to the same affiliation, they tend to connect to each other. For example, people of the same department interact with each other more frequently than any two random people in a network. In order to infer the latent affiliations, we need to find out a group of people who interact with each other more frequently than random. This boils down to a classical community detection problem in networks. Most existing community detection methods partition the nodes of a network into disjoint sets. But in reality, each actor is likely to subscribe to more than one affiliation. Henceforth, a soft clustering scheme is preferred to extract social dimensions.

Many approaches developed for clustering on graphs serve the purpose of social dimension extraction, including modularity maximization [101], latent space models [57, 110], block models [105, 3] and spectral clustering [88]. In Appendix A, we show that all these methods can follow a similar procedure for community detection as shown in Figure A.2. Given a network, we construct

Table 2.2: Social Dimensions Extracted According to Spectral Clustering

Node	Ideal Case		Spectral Clustering
1	1	1	-0.0893
2	1	0	0.2748
3	0	1	-0.4552
4	0	1	-0.4552
5	0	1	-0.4643
6	1	0	0.1864
7	1	0	0.2415
8	1	0	0.3144
9	1	0	0.3077

a utility matrix, extract its top (largest or smallest depending on the objective function) eigenvectors as the soft community indicator, and then recover community partition via k-means clustering. Since a soft clustering is preferred for community detection, we use the soft community indicator as social dimensions.

Take spectral clustering as an example. Spectral clustering is originally proposed to address the partition of nodes in a graph. Spectral clustering has been shown to work reasonably well in various domains including graphs, text, images and microarray data. It is also proved [157] to be equivalent to a soft version of the classical k-means algorithm for clustering. Social dimensions in this case correspond to the smallest eigenvector of the utility matrix, i.e., the graph Laplacian defined in Eq. (A.9). We want to emphasize that spectral clustering is not the only method of choice. Different alternatives can be plugged in for this phase. This is also one nice feature of our framework as it allows for convenient plug-in of existing soft clustering packages.

For example, if we apply spectral clustering to the toy network in Figure 2.2, we obtain the social dimension in the last column in Table 2.2. In the table, we also show the ideal representation of the two affiliations in the network. Nodes 3, 4 and 5 belong to the same affiliation, thus share similar negative values in the extracted social dimension. Nodes 2, 6, 7, 8 and 9, associated with Affiliation-1, have similar positive values. Node 1, which bridges these two affiliations, has a value in between. The social dimension extracted based on spectral clustering do capture actor affiliations in certain degree.

In summary, given a network A, we construct the normalized graph Laplacian $\widetilde{L}$ as in Eq. (A.9), and then compute its first k smallest eigenvectors as the nodes' social dimensions. Note that $\widetilde{L}$ is sparse. So the power method or Lanczos method [50] can be used to calculate the top eigenvectors if k is not too large. Many existing numerical optimization software can be employed.

2.3.2 Phase II: Classification Learning based on Social Dimensions

This phase constructs a classifier with the following inputs:

- the labels $\mathbf{Y}^L$ of labeled nodes in the network $\mathscr{A}$,
- the social dimensions S^L of the labeled nodes.

The social dimensions extracted in the first phase are deemed as features of data instances (nodes). We conduct conventional supervised learning based on the social dimensions and the label information. A discriminative classifier like support vector machine (SVM) or logistic regression can be used. Other features associated with the nodes, if available, can also be included during the discriminative learning. This phase is critical as the classifier will determine which dimensions are relevant to the class label. A linear SVM is exploited due to its simplicity and scalability [128].

2.3.3 Prediction

The prediction phase requires:

- The constructed classifier based on training,
- The social dimensions S^U of those unlabeled nodes in the network.

Prediction is straightforward once the classifier is ready, because social dimensions have been calculated in Phase I for all the nodes, including the unlabeled ones. We treat social dimensions of the unlabeled nodes as features and apply the constructed classifier to make predictions. Different from existing within-network classification methods, collective inference becomes unnecessary. Though the distribution of actors does not follow the conventional i.i.d. assumption, the extracted social dimensions in Phase I already encode correlations between actors along with the network. Each node can be predicted independently without collective inference. Hence, this framework is efficient in terms of prediction.

We emphasize that this proposed framework SocioDim is flexible. We choose spectral clustering to extract social dimensions and SVM to build the classifier. This does not refrain us from using alternative choices. Any soft clustering scheme can be used to extract social dimensions in the first phase. The classification learning phase can also be replaced with any classifier other than SVM. This flexibility enables the immediate use of many existing software packages developed for clustering or classification.

2.4 Experiment Setup

In the experiment, we will compare our proposed SocioDim framework with representative collective-inference methods when heterogeneity is present in a network. Before we proceed to the details of experiments, we describe the data collected for experiments and baseline methods for comparison.

2.4.1 Data Sets

We focus on classification tasks specifically in social media. We shall examine how different approaches behave on real-world social networks. Two data sets are collected: one from BlogCatalog[1] and the other from a popular photo sharing site Flickr[2]:

- **BlogCatalog** is a blog directory that hosts various information pieces like the categories a blog is listed under, blog level tags, snippets of 5 most recent blog posts, and blog post level tags. Bloggers submit their blogs to BlogCatalog and specify the metadata mentioned above for improved access to their blogs. This way the blog sites are organized under pre-specified categories. A blogger also specifies his social network with other bloggers. A blogger's interests could be gauged by the categories he publishes his blogs in. Each blogger could list his blog under more than one category. Note that we only crawl a small portion of the whole network. Some categories occur rarely and they demonstrate no positive correlation between neighboring nodes in the network. Thus, we pick 39 categories with a reasonably large sample pool for evaluation purpose. On average, each blogger lists their blog under 1.6 categories.

[1] http://www.blogcatalog.com/
[2] http://www.flickr.com/

Table 2.3: Statistics of Social Media Data

Data	BlogCatalog	Flickr
Categories (K)	39	195
Actors (n)	10, 312	80, 513
Links (m)	333, 983	5, 899, 882
Network Density	6.3×10^{-3}	1.8×10^{-3}
Maximum Degree	3, 992	5,706
Average Degree	65	146
Average Labels	1.4	1.3
Category Normalized Cut	0.48	0.46

- **Flickr** is a popular website to host personal photos uploaded by users, and also an online community platform. Users in Flickr can tag photos and add contacts. Users can also subscribe to different interest groups ranging from *black and white photos*[3] to a specific subject (say *bacon*[4]). Among the huge network and numerous groups collected from Flickr, we randomly pick around 200 interest groups as the class labels and crawl the contact network among the users subscribed to these groups for our experiment. The users with only one single connection are removed from the data set.

Table 2.3 lists some statistics of the network data. As seen in the table, the connections among social actors are extremely sparse. The degree distribution is highly imbalanced, a typical phenomenon in scale-free networks. We also compute the normalized cut score (see Eq. (A.7)) for each category and report the average score. Note that if a category is nearly-isolated from the rest of a network, the normalized cut score should be close to 0. Clearly, for both data sets, the categories are not well-separated from the rest of the network, implying the difficulty of the classification tasks. Both data sets are publicly available from the first author's homepage.

2.4.2 Baseline Methods

We apply SocioDim to both data sets. Spectral clustering is employed to extract social dimensions. The number of latent dimensions is set to 500 and one-vs-rest linear SVM is used for discriminative learning. We also compare SocioDim to two representative relational learning methods based

[3]http://www.flickr.com/groups/blackandwhite/
[4]http://www.flickr.com/groups/everythingsbetterwithbacon/

on collective inference (Weighted-Vote Relational Neighbor Classifier [89] and Link-Based Classifier [87]), and two baseline methods without learning (a Majority Model and a Random Model):

- Weighted-Vote Relational Neighbor Classifier (wvRN). wvRN [89] works like a lazy learner. No learning is conducted during training. In prediction, the relational classifier estimates the class membership $p(\mathbf{y}_i|\mathcal{N}_i)$ as the weighted mean of its neighbors.

$$p(\mathbf{y}_i|\mathcal{N}_i) = \frac{1}{\sum_j w_{ij}} \sum_{v_j \in \mathcal{N}_i} w_{ij} \cdot p(\mathbf{y}_j|\mathcal{N}_j) \tag{2.3}$$

$$= \frac{1}{|\mathcal{N}_i|} \sum_{\{j:(v_i,v_j)\in E\}} p(\mathbf{y}_j|\mathcal{N}_j) \tag{2.4}$$

  where w_{ij} in (2.3) are the weights associated with the edge between node v_i and v_j. Eq. (2.4) is derived, because the networks studied here use $\{0,1\}$ to represent connections between actors and we only consider the first order Markov assumption (The labels of one actor depend on his connected friends). Collective inference is exploited for prediction. We iterate over each node of the network to predict its class membership until the change of all the nodes is small enough. wvRN has been shown to work reasonably well for classification in the univariate case. It is recommended as a baseline method for comparison [90].

- Network Only Link-Based Classifier (LBC) [87]. This classifier creates relational features of one node by aggregating the label information of its neighbors. Then a relational classifier can be constructed based on labeled data. In particular, we use averaged class membership (as in Eq. (2.4)) of each class as relational features, and employ SVM to build the relational classifier. For prediction, relaxation labeling [25] is utilized as the collective inference scheme.

- Majority Model (MAJORITY). This baseline method uses the label information only. It does not leverage any network information for learning or inference. It simply predicts the class membership as the proportion of positive instances in the labeled data. All nodes are assigned the same class membership. This model is inclined to predict categories of larger size.

- Random Model (RANDOM). As indicated by the name, this model predicts the class membership for each node randomly. Neither network nor label information is used. This model is included for relative comparison of various methods.

2.4.3 Evaluation Measure

In our experiments, actors might have more than one label. We apply the methods above to each category independently and report the average performance. Since most methods yield a ranking of labels rather than an exact assignment, a thresholding process is normally required. It has been shown that different thresholding strategies lead to quite different performances [38, 128]. To avoid the effect of thresholding, we assume the number of labels on the test data is already known, and check how the top-ranking predictions match with the true labels. Precision and recall are widely used criterion to evaluate the performance. Typically, a classifier finds a tradeoff between precision and recall. Another commonly used criterion is F-measure, i.e., the harmonic mean of precision and recall. As we have multiple labels, both Micro-F1 and Macro-F1 [38, 128] are adopted to evaluate the classification performance.

Given test data $\mathbf{X} \in R^{N \times M}$, let $\mathbf{y_i}, \hat{\mathbf{y}}_\mathbf{i} \in \{0,1\}^K$ be the true label set and the predicted label set for instance $\mathbf{x_i}$. Macro-F1 is the F1 averaged over categories.

$$M\text{acro-F1} = \frac{1}{K}\sum_{k=1}^{K} F_1^k \tag{2.5}$$

For a category C_k, the precision (P^k) and the recall (R^k) are calculated as,

$$P^k = \frac{\sum_{i=1}^{N} y_i^k \hat{y}_i^k}{\sum_{i=1}^{N} \hat{y}_i^k}; \quad R^k = \frac{\sum_{i=1}^{N} y_i^k \hat{y}_i^k}{\sum_{i=1}^{N} y_i^k}.$$

Then F1 measure, defined as the harmonic mean of precision and recall is computed as follows:

$$F_1^k = \frac{2P^kR^k}{P^k + R^k} = \frac{2\sum_{i=1}^{N} y_i^k \hat{y}_i^k}{\sum_{i=1}^{N} y_i^k + \sum_{i=1}^{N} \hat{y}_i^k}$$

Micro-F1 is computed using the equation of F_1^k and considering the predictions as a whole. More specifically, it is defined as

$$\textit{Micro-F1} = \frac{2\sum_{k=1}^{K}\sum_{i=1}^{N} y_i^k \hat{y}_i^k}{\sum_{k=1}^{K}\sum_{i=1}^{N} y_i^k + \sum_{k=1}^{K}\sum_{i=1}^{N} \hat{y}_i^k}. \tag{2.6}$$

According to the definition, macro-F1 is more sensitive to the performance of rare categories while micro-F1 is affected more by the major categories. In our experiments, both measures are examined carefully.

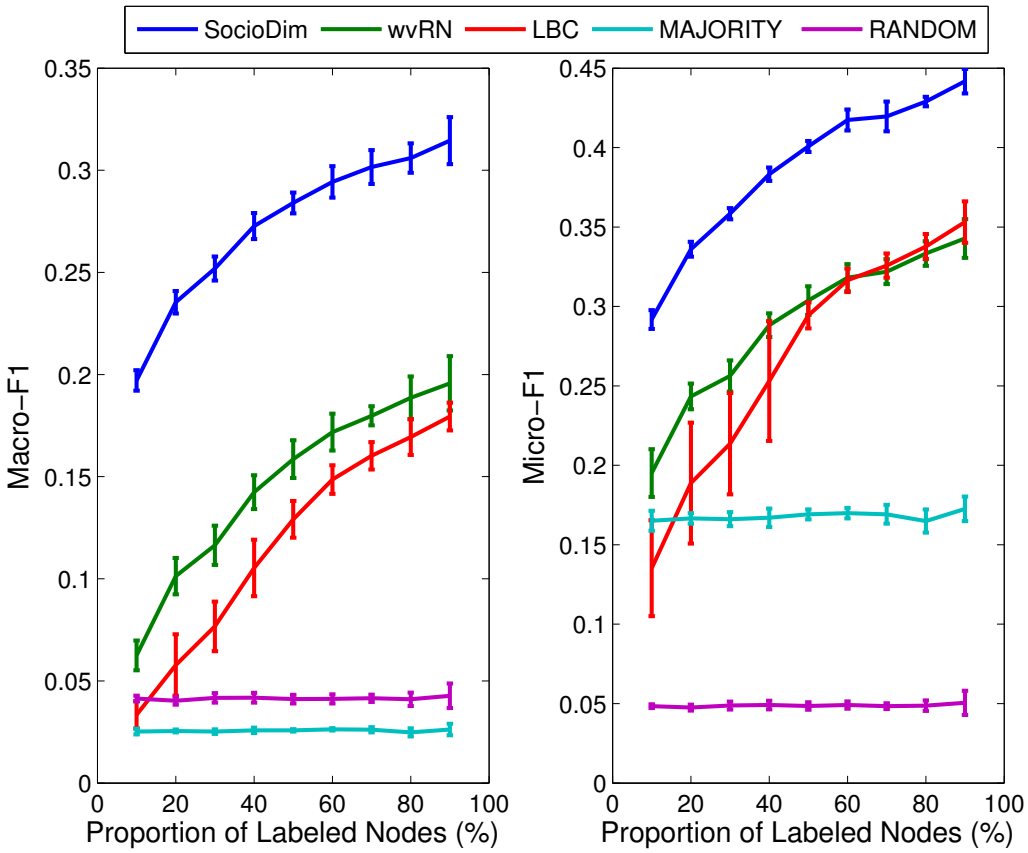


Figure 2.5: Performance on BlogCatalog with 10,312 Nodes (Better viewed in color)

2.5 Experiment Results

In this section, we will experimentally examine the following questions: How is the classification performance of our proposed framework compared to that of collective inference? Does differentiating heterogeneous connections presented in a network help yield a better performance?

2.5.1 Prediction Accuracy on BlogCatalog Data

We gradually increase the number of labeled nodes from 10% to 90%. For each setting, we randomly sample a portion of nodes as labeled. This process is repeated 10 times and the average performance are recorded. The performances of different methods and the standard deviation are plotted in Figure 2.5. Clearly, our proposed SocioDim outperforms all the other methods. wvRN, as shown in the figure, is the runner-up most of the time. MAJORITY performs even worse than RANDOM in terms of Macro-F1 as it always picks the majority class for prediction.

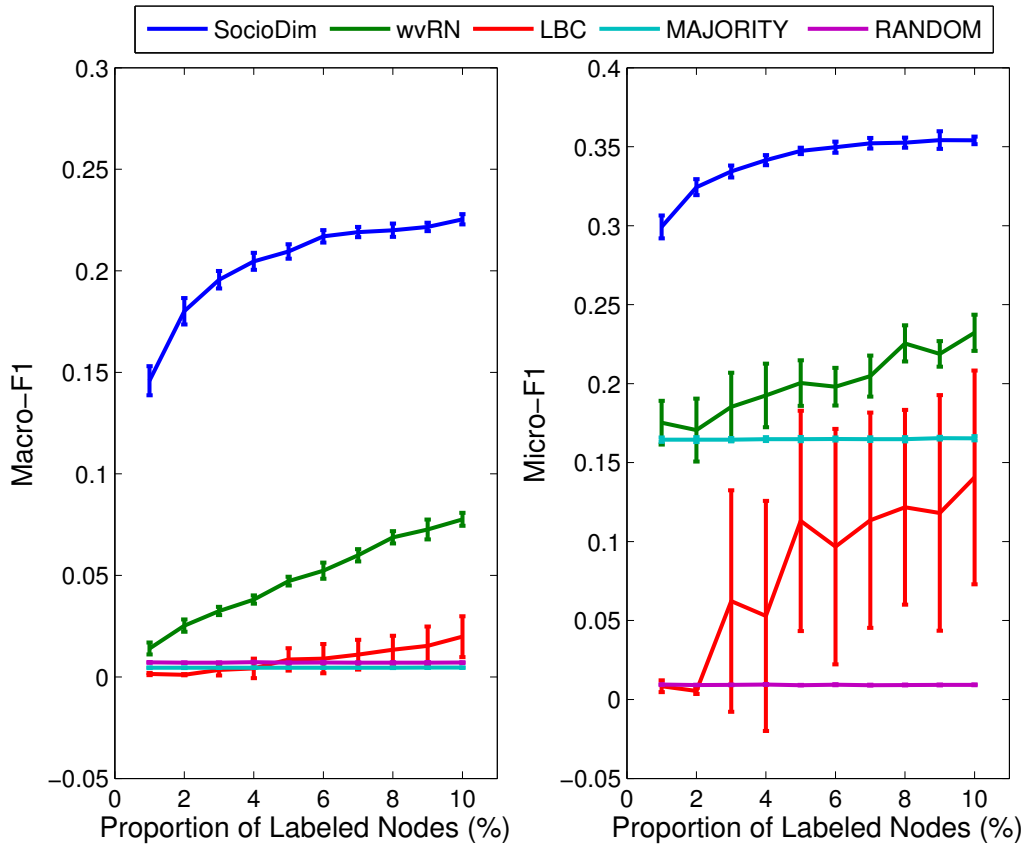


Figure 2.6: Performance on Flickr with 80,513 Nodes (Better viewed in color)

The superiority of SocioDim over other relational learning methods with collective inference is evident. As shown in the figure, the link based classifier (LBC) performs poorly with few labeled data. This is because LBC requires to learn a relational classifier on labeled data before the inference. When samples are few, the learned classifier is not robust enough. This is indicated by the large deviation of LBC in the figure when labeled samples are less than 50%. We notice that LBC in this case takes many iterations to converge. wvRN is more stable, but its performance is not comparable to SocioDim. Even with 90% of nodes being labeled, a substantial difference between wvRN and SocioDim is still observed. Comparing all the three methods (SocioDim, wvRN and LBC), SocioDim is most stable and achieves the best performance. A similar trend is observed for both precision and recall as well.

2.5.2 Prediction Accuracy on Flickr Data

Compared with BlogCatalog, the Flickr data is on a larger scale, with around 100,000 nodes. In practice, the label information in large-scale networks is often very limited. Here we examine a similar case. We change the proportion of labeled nodes from 1% to 10%. Roughly, the number of labeled actors increases from around $1,000$ to $10,000$. The performances are reported in Figure 2.6.

The methods based on collective inference, such as wvRN and LBC, perform poorly. The LBC fails most of the time (almost like random) and is highly unstable. This can be verified by the fluctuation of Micro-F1 of LBC. LBC tries to learn a classifier based on the features aggregated from a node's neighbors. The classifier can be problematic when the labeled data are extremely sparse and the network is noisy as presented here. While alternative collective inference methods fail, SocioDim performs consistently better than other methods by differentiating heterogeneous connections in the network.

It is noticed the prediction performance on both data sets is around 20-30% for F1-measure, suggesting that social media networks are very noisy. As shown in later experiments, the performance can be improved when other actor features are also included for learning.

2.5.3 Efficiency Comparison

In social media, the populace involved are normally immense. The scalability and efficiency of different methods should be considered carefully for practical deployment. Thus, we examine the efficiency of our proposed framework with wvRN, the runner-up in terms of classification performance. Table 2.4 lists the computation time of pre-processing, training and prediction for both methods on the Flickr data set as measured by a Core2Duo E8400 CPU.

Our proposed framework SocioDim is computationally expensive in data pre-processing and training. The major computational burden lies on the extraction of social dimensions from a given network. For the Flickr data, it takes almost 50 minutes to extract 500 social dimensions. This step does not require the label information. Hence it is considered as pre-processing step and can be finished before learning the classifier. Methods based on collective inference, on the other hand, are normally quite efficient for training. The method wvRN is a lazy learner. No computation is involved for pre-processing and training but it is very slow during testing, thus wvRN does not show

Table 2.4: Computation Time of Different Methods on Flickr in terms of Seconds

Proportion of Labeled Nodes		1%	2%	3%	4%	5%	6%	7%	8%	9%
Pre-Processing	SocioDim	2857								
Training	SocioDim	29.5	46.5	69.1	91.5	118.5	134.1	154.5	162.6	203.8
Testing	SocioDim	2.5	2.4	2.4	2.4	2.4	2.3	2.5	2.3	2.3
	wvRN	1247	1387	1268	1084	860	740	646	588	502

up in the table in the first two rows.

As for prediction, SocioDim is clearly the winner as presented in the table. Since social dimensions for all the nodes are already extracted in the pre-processing step, we can apply the constructed classifier to the extracted dimensions of the unlabeled nodes directly to make predictions. Collective inference, on the contrary, normally requires multiple scans of the network until convergence. It is not surprising that SocioDim takes 2 seconds to predict while wvRN requires hundreds or thousands more time to complete the task. A majority of real-world applications and online services in social media, weigh much more on prediction time, rather than training time. Hence, SocioDim is preferred with respect to prediction efficiency.

2.5.4 Understanding SocioDim Framework

In the previous subsections, we show that SocioDim outperforms representative methods based on collective inference. *Why does SocioDim demonstrate better performance over collective inference?* We will explore further different hypotheses to better understand the SocioDim framework.

H_1: Does SocioDim win because a given network is too sparse? One might suspect that the poor performance of collective inference is due to the sparsity of a given network. As shown in Table 4.3, the density of the BlogCatalog network is only 6.3×10^{-3}. Flickr is even sparser. When a network is too sparse, Gallagher et al. [44] suggest expanding the neighborhood of one node by adding "ghost edges" to connect those nodes that are 2-hop away. Following their idea, we construct a network by linking all nodes that are within 2 hops. After the expansion for BlogCatalog, the network density leaps to 6.16×10^{-1}. We cannot expand any more as the network becomes almost a complete graph when nodes within 3 hops are connected. Flickr becomes quite dense after a 2-hop expansion, causing computational problems. Therefore, we report the performance of collective inference of

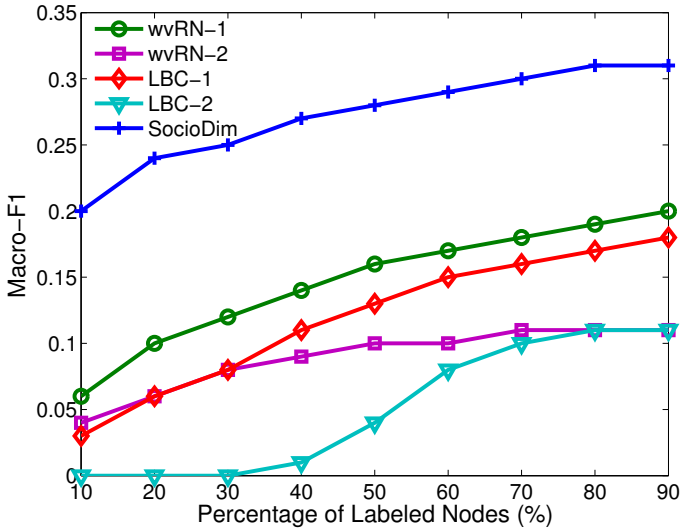


Figure 2.7: Performances of Collective Inference by Expanding the Neighborhood

the expanded network on BlogCatalog only in Figure 2.7, where i denotes the number of hops to consider for defining neighborhood. For both wvRN and LBC, the performance deteriorates after the neighborhood is expanded. This is because the increase of connections, though alleviating the sparsity problem, seems to introduce more heterogeneity. Collective inference, no matter how we define the neighborhood, is not comparable to SocioDim.

H_2: Does SocioDim win because nodes within a category are well isolated from the rest of a network? Another hypothesis is related to the community effect presented in a network. Since SocioDim relies on soft community detection to extract social dimensions and the data we studied are extremely sparse, one might suspect SocioDim wins because there are very few inter-category edges. Intuitively, when a category is well isolated from the whole network, the clustering in the first Phase of SocioDim captures this structure, thus it defeats collective inference. Surprisingly, *this intuition is not correct*. Based on our empirical observation, when a category is well isolated, there is not much difference between wvRN and SocioDim. SocioDim's superiority is more observable when the nodes of one category are blended into the whole network.

In order to calibrate whether the nodes of one category are isolated from the rest of a given network, we compute the *category normalized cut* (NCut) following Eq. (A.7). Given a category,

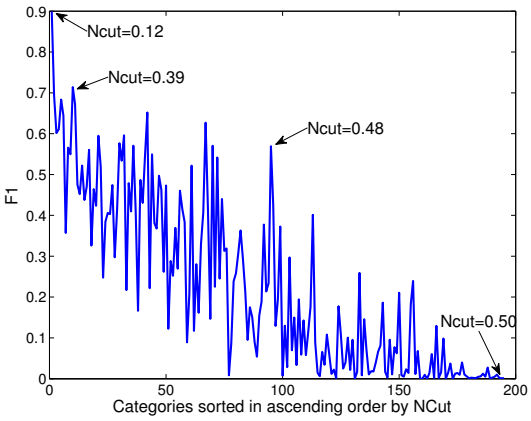


Figure 2.8: Performance of SocioDim of Individual Categories

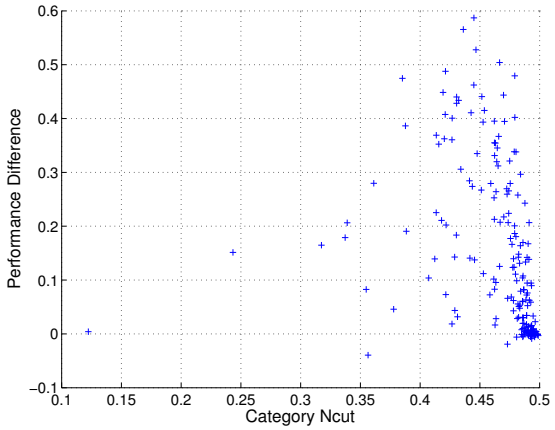


Figure 2.9: Performance Improvement of SocioDim over wvRN wrt. Category Ncut

we split a network into two sets: one set containing all the nodes of the category, the other set all the nodes not belonging to that category. The normalized cut can be computed. If a category is well-isolated from a network, the Ncut should be close to 0. The larger the NCut is, the less the category is isolated from the remaining network. The average category Ncut scores on BlogCatalog and Flickr are 0.48 and 0.46 respectively as reported in Table 2.3. This implies that most categories are actually well connected to the remaining network, rather than being an isolated group as one supposes.

Figure 2.8 shows the performance of SocioDim on the 195 individual categories of the Flickr data when 90% of nodes are labeled. As expected, SocioDim performance tends to decrease when the Ncut increases. An interesting pattern emerges when we plot the improvement of SocioDim over wvRN with respect to category Ncut in Figure 2.9. The performance improvement multiplies when Ncut increases. Notice the plus at the bottom left, which corresponds to the case when $Ncut = 0.12$. For this category, SocioDim achieves 90% F1 as shown in Figure 2.9. But the improvement over wvRN is almost 0 as in Figure 2.9. That is, *wvRN* is comparable. Most of SocioDim's improvements occur when $Ncut > 0.35$. Essentially, when a category is not well-isolated from the remaining network, SocioDim tends to outperform wvRN substantially.

H_3: Does SocioDim win because it addresses network heterogeneity? In the previous subsection, we notice that SocioDim performs considerably better than wvRN when a category is not so well

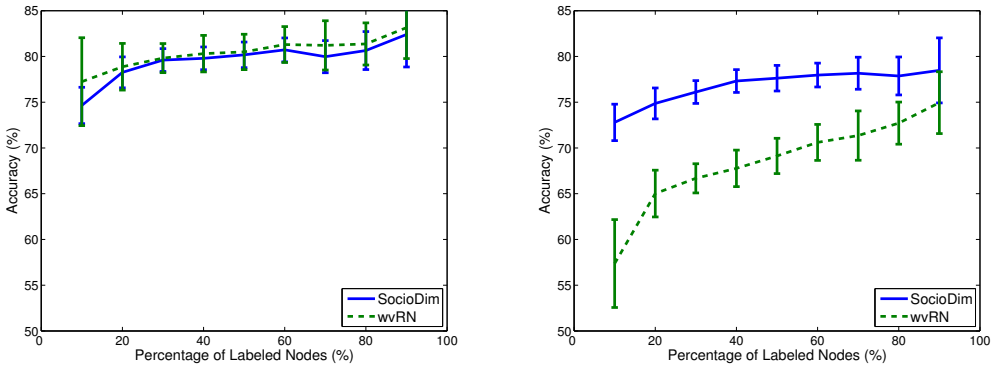
Table 2.5: Statistics of *imdb* Data

Data	Size	Density	Ave. Degree	Base Acc.	Category Ncut
$imdb_{prodco}$	1126	0.035	38.8	0.501	0.24
$imdb_{all}$	1371	0.049	67.3	0.562	0.36

separated. We attribute the gain to taking into account connection heterogeneity. Hence, we adopt a benchmark relational data (*imdb* used in [90]) with varying heterogeneity. The *imdb* network is collected from the Internet Movie Data base, with nodes representing 1377 movies released between 1996 and 2001. The goal is to estimate whether the opening weekend box-office receipts of a movie "will" exceed 2 million. Two versions of network data are constructed: $imdb_{prodco}$ and $imdb_{all}$. In $imdb_{prodco}$, two movies are connected if they share a production company. While in $imdb_{all}$, two are connected if they share actors, directors, producers or production companies. Clearly, the connections in $imdb_{all}$ are more heterogeneous. Both network data sets have one giant connected component each, with others being singletons or trivial-size components. Here we report the performance on the largest components.

We notice that these two data sets demonstrate different characteristics from the previously-studied social media data: 1) the connections are denser. For instance, the density of $imdb_{all}$ is 0.049 (7 times denser than BlogCatalog and 27 times than Flickr); 2) the classification task is also much easier. It is a binary classification task. The class distribution is balanced, different from the imbalanced distribution present in the social media data. Hence, we report classification performance in terms of accuracy as in [90]; and 3) classes are well separated as suggested by the low category Ncut in Table 2.5.

Figure 2.10 plots the performance of SocioDim and wvRN on $imdb_{prodco}$ and $imdb_{all}$. When connections are relatively homogeneous (e.g., $imdb_{prodco}$ data), SocioDim and wvRN demonstrate comparable classification performance, wvRN being slightly better by 1%. When connections become heterogeneous, the category NCut increases from 0.24 to 0.36. Both methods' performance decreases as shown in Figure 2.10b. For instance, with 50% of nodes are labeled, SocioDim's accuracy decreases from 80% to about 77%, but wvRN's performance drops severely, from 80% to around 66% with introduced heterogeneity.



(a) $imdb_{prodco}$ (b) $imdb_{all}$

Figure 2.10: Classification Performance on *imdb* Network

We notice that the performance decrease of wvRN is most observable when labeled data are few. With increasing available labeled data, wvRN's performance climbs up. The comparison on the two networks of distinctive degree of heterogeneity confirms our original hypothesis: *SocioDim, by differentiating heterogeneous connections, performs better than collective inference*. This effect is more observable when a network presents heterogeneity and labeled data are few.

2.5.5 *Visualization of Extracted Social Dimensions*

In order to get some tangible idea of extracted social dimensions, we examine tags associated with each dimension. It is impractical to show tag clouds of all the extracted dimensions (500 social dimensions for both data sets). Thus, given a category, we investigate its dimension with the maximum SVM weight, and check whether it is really informative of the category.

A minor issue is that each dimension is represented by continuous values (as in the last column in Table 2.2), because we use soft clustering to extract social dimensions. For simplicity, we pick the top 20 nodes with the maximum positive values as representatives of one dimension. For example, nodes 8, 9, 7 and 6 are the top 4 nodes for the social dimension extracted following spectral clustering in Table 2.2. Tags of those representative nodes in one dimension are aggregated as the tags of that dimension. For the sake of clear visualization, only the tags occurring more than once are presented as in a tag cloud with font size denoting their relative frequency.

Here we just showcase some examples from BlogCatalog. Figure 2.11 lists the tag clouds of selected social dimensions for categories *Health* and *Animal*, respectively. In the clouds, the

(a) Health (b) Animal

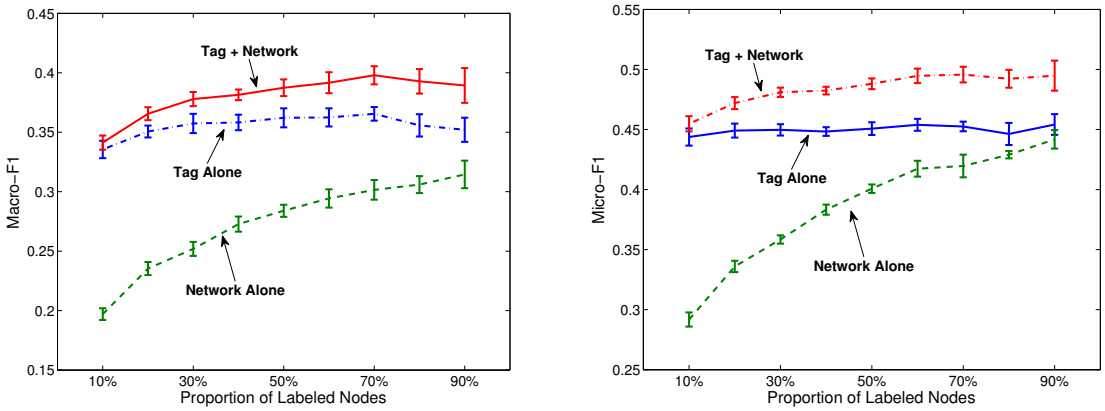
Figure 2.11: Social dimensions Selected by *Health* and *Animal*

position and direction of tags do not matter. Only the font size are meaningful, represent the relative frequency of each tag. Clearly, both are quite relevant to their target categories. Based on the tag cloud in Figure 2.11a, it is not difficult to figure out that the social dimension is about food and weight loss, which is highly relevant to *Health*. Similarly, the social dimension in Figure 2.11b is about dogs and pets, thus relevant to *Animal*. These examples suggest that our extracted social dimension are sensible, and relevant dimensions can be selected accordingly by the classification learning phase of the SocioDim framework.

2.5.6 Integration of Actor Network and Actor Features

In social media, various kinds of user information besides social networks can be collected. For instance, in blogosphere, people post blogs, write comments and upload tags. Some users also provide some profile information. It is desirable to utilize all the information available to achieve more accurate classification. However, the actor features (e.g., user profiles, social content or tag information) and the networks are presented in disparate formats, hence some efforts are required for the integration.

One nice property of SocioDim is that it converts a network into features. Thus, if actor features are available, it is straightforward to couple the network features with actor features: simply combine the extracted social dimensions with actor features, and let the discriminative learning procedure to determine which features are more informative of a class label. This simple combination of network information and actor features allows for integration of data in disparate format and can lead to more accurate classification in general. Here we take BlogCatalog as an example to show the



(a) Macro-F1 (b) Micro-F1

Figure 2.12: Performance of Network with Actor Features on BlogCatalog

effect. In BlogCatalog, the blogger can upload some tags of his blog site. We use tag information as actor features for the bloggers. The performance of using tag or network alone, or the combination of the two are plotted in Figure 2.12.

Tags are normally quite descriptive of a blogger while networks tend to be noisy. It should not be surprising that the performance based on tags alone is better than the performance based on networks. It is noticed that increasing the labeled samples does not help much for performance based on tags, because some users do not provide tags. But if we combine the social dimensions extracted from a network with the tag features, the performance is increased by 3-6%. The network in social media is noisy. It provides complementary, though maybe weak, information of user interests. There are other relational models to capture the dependency of connected nodes and additional attributes (e.g.,[136, 135]), but they normally require a lot of efforts. Our proposed SocioDim provides a *simple* yet effective approach to integrate network information and actor features for accurate classification.

2.6 Related Work

As our SocioDim framework addresses within-network classification, which is a special case of collective classification and semi-supervised learning, we review literature about these two fields respectively.

2.6.1 Collective Classification

Collective classification [112] refers to the classification when objects or entities are presented in multiple relations or network format. In this work, we study a special case: within-network classification [90] when the objects are connected in one network. The data instances in the network are not independently identically distributed (i.i.d.) as in conventional data mining. In order to capture the correlation between labels of neighboring data objects, a Markov dependency assumption is widely adopted. That is, the labels of one node depend on the labels (and attributes) of its neighbors. Based on the assumption, collective inference [64] is proposed for prediction. Normally, a relational classifier is constructed based on the relational features of labeled data, and then an iterative process is required to determine class labels for unlabeled data. It is shown that a simple weighted vote relational neighborhood classifier [89] works reasonably well on some benchmark relational data and is recommended as a baseline for comparison [90].

In our implementation of collective inference, we define the neighborhood to be the nodes that are only 1-hop away. Gallagher et al. [44] propose to add "ghost edges" before relational learning when the network is too sparse. The ghost edges essentially connect nodes that are 2 hops away. After the expansion of the neighborhood of one node for collective inference, they observe a better classification performance. However, this strategy cannot be applied to networks in social media. In social networks, the small-world effect [140] is often observed [23]. That is, any pair of nodes in a large-scale social network are only several hops away, relating to the well-known "six degree of separation". For instance, in our Flickr data, the average degree of one node is 146. Roughly, the nodes that are two hops away from one node can be as high as $146 \times 146 = 21,316$. Of course, this number is not precise as the friends of friends may overlap. This huge number of neighbors brings in much more noise and heterogeneity in connections, which can worsen the performance of collective inference. This is empirically verified in a smaller BlogCatalog network in Section 2.5.4. Often, a network becomes very dense after neighborhood expansion. As a result, the scalability can also be a concern.

There are many more complicated relational models to model the dependence between connected entities. For instance, probabilistic relational model (PRM) as introduced in [136, 135].

Please refer to [47] for a comprehensive treatment. No doubt that such models are quite powerful to model various dependencies amongst entities, though the subsequent inference always requires certain approximation. Their complexity and scalability are often a barrier for practical use. Indeed, Macskassy and Provost compared wvRN with PRM, and found that wvRN outperforms PRM on several relational data sets [90]. Given the extreme simplicity of wvRN and its outstanding performance, wvRN is adopted as a baseline in our experiments.

Many relational classifiers only capture the local dependency based on the Markov assumption. To capture the long-distance correlation, the latent group model [98] and the nonparametric infinite hidden relational model [152] are presented. Both present generative models such that the links (and actor attributes) are generated based on actors' latent cluster membership. They share a similar spirit as SocioDim. But the model intricacy and high computational cost for inference hinders their direct application to huge networks. So Neville and Jensen in [98] propose to use a clustering algorithm to find the hard cluster membership of each actor first, and then fix the latent group variables for later inference. In social media, a network is often very noisy. Some nodes do not show a strong community membership and hard clustering might assign them randomly [58]. The resultant community structure can change drastically even with the removal of one single edge in the network. Our social dimensions are represented as continuous values. Each node is allowed to be involved in different dimensions in a flexible manner. It is also empirically verified that hard partition is not comparable to soft clustering as shown in [121]. Another difference is that both latent group model and nonparametric infinite hidden relational model are generative, while SocioDim allows the plug-in of discriminative classifier. In conjunction with the discriminative power of SVM, SocioDim yields more accurate and stable performances.

Recently, Neville [97] suggested that simple random sampling for cross validation on network data tends to yield elevated Type-I error when comparing two algorithms. That is, such an evaluation scheme might report two algorithms to be significantly different when the two are indeed similar. A refined network cross validation scheme is proposed. We hope to evaluate our algorithm following the new evaluation strategy as well.

2.6.2 Semi-Supervised Learning

Another related field is semi-supervised learning [162]. Semi-supervised learning is original proposed to address the label shortage problem by exploiting unlabeled data. One branch of semi-supervised learning is the graph-based approach [163, 158]. Indeed, they share quite a similar assumption as collective inference. The performances of wvRN and Zhu's method [163] are nearly identical as reported in [90]. Considering that Zhu's method involves the computation of the inverse of a matrix of the same size as a given network, wvRN is used as the baseline in our experiments.

Some work [82, 26] attempts to address semi-supervised learning with multiple labels by utilizing the relationship between different labels. The relationship can be obtained either from external experts or computed based on the labeled data. But its computational cost is prohibitive. We tried the method presented in [26], which constructs a graph between different labels and then compute a label assignment such that it is smooth on both the instance graph and the label graph. It requires to solve a Sylvester equation [50] and direct implementation takes extremely long time to find a solution, preventing us from reporting any comparative results.

On the other hand, some papers try to construct kernels based on graphs for SVM. Diffusion kernel [71] is a commonly used one. However, it requires full SVD of the graph Laplacian, which is not applicable for large-scale networks. Empirically, the classification performance is sensitive to the diffusion parameter. Cross validation or some variant of kernel learning is required to select a proper diffusion kernel [143].

2.7 Summary

Social media provides a virtual social networking environment. The presence of partial label information and networking information allows us to build better classifiers. This work proposes a novel approach to deal with heterogeneous connections prevalent in social media. To differentiate heterogeneous connections, we propose to extract latent social dimensions via soft clustering such as modularity maximization and spectral clustering. Based on the extracted social dimensions, a discriminative classifier like SVM can be constructed to determine which dimensions are informative for classification. Extensive experiments on social media data demonstrated that our proposed social dimension approach outperforms alternative relational learning methods based on collective

inference, especially when labeled data are few. It is noticed that some relational models perform poorly in social media data. This is due to the heterogeneity of connections and high irregularity of human interactions in social media. Our approach, by differentiating connections among social actors and converting network information into conventional features, achieves effective learning for classification. Of course, our proposed learning framework is not confined to social media only. In other domains, such as cellular networks, email communications, traditional Web and Internet, SocioDim might also demonstrate its power.

Chapter 3

UNSUPERVISED LEARNING WITH SOCIAL MEDIA NETWORKS

A *community* (or *group*) is a set of users that interact with each other frequently [147]. Unsupervised learning with social media networks refers to community detection. It has a broad range of applications to discover groups. For example, some communities can be extracted as social dimensions for supervised learning as presented in last chapter. Other applications include network visualization, intelligence analysis [10], network compression [116], behavioral study [56], influence modeling [1], and collaborative filtering [27]. A variety of community detection (a.k.a. finding cohesive subgroups [147]) methods have been proposed to capture such social structures in a network. However, some new properties prevail in social media networks, requiring novel techniques. In this chapter, we show that the social-dimension based learning framework proposed in the previous chapter can be adapted to handle unsupervised learning with social media networks. We will first discuss several types of networks in social media, and then present techniques to extract communities for each type of networks. It turns out that all the methods can be nicely covered by the social-dimension based learning framework.

3.1 Types of Social Media Networks

Social media enables rich interaction between users. Networks in social media can consist of heterogeneous types of interactions or entities. They can also be highly dynamic with interaction changes. Accordingly, social media networks can be categorized into the following: multi-dimensional networks [129], multi-modal networks [127] or dynamic networks with evolutions.

- *Multi-dimensional networks*. Communications in social media are multi-dimensional. Each dimension represents one type of activity between users. A multi-dimensional network has multiple types of interactions between the same set of users. For instance, in Figure 3.1, at popular photo and video sharing sites (e.g., Flickr and YouTube), a user can connect to his friends through email invitation or the provided "add as contacts" function; users can also tag/comment on the social contents like photos and videos; a user at YouTube can respond to another user by uploading a video; and a user can also become a fan of another user by

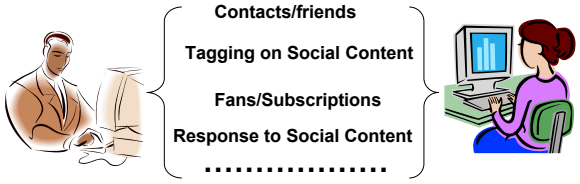


Figure 3.1: Communications in Social Media are Multi-Dimensional

subscription to the user's contributions of social contents. A network among these users can be constructed based on each form of activity, in which each dimension represents one type of interaction.

- *Multi-modal Networks*. A multi-modal network (a.k.a. multi-mode network, multi-type relational network) [84, 127] involves heterogeneous actors. Each mode represents one type of entity. For instance, in the YouTube example above, a 3-mode network can be constructed, with videos, tags and users each representing a mode, as seen in Figure 3.2. There are disparate interactions among the three types of entities: users can upload videos. They can also provide tags for some videos. Intuitively, two users contributing similar videos or tags are likely to share interests. Videos sharing similar tags or users are more likely to be related. Note that in the network, both tags, and videos are also considered as "actors", though users are probably the major mode under consideration.

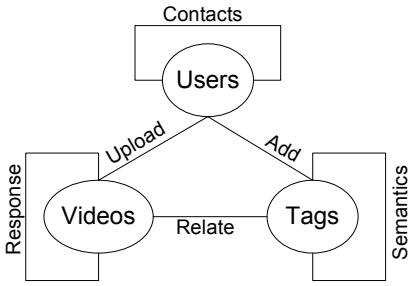


Figure 3.2: An example of 3-Mode Network in YouTube

- *Dynamic networks*. In social media, networks are highly dynamic. Each day, new members might join a network and new connections are established. Some existing members might become dormant as well. This yields a highly dynamic network. Kumar et al. [73] show that the number of communities and some other network statistics such as degrees and size of

the maximum connected component observe a significant change around 2001 in the blogosphere. Some social media sites like Facebook and Twitter has been observing a tremendous growth in recent years. As for a dynamic network, we obtain a sequence of similar yet evolving observations if we take multiple snapshots.

With the growth of networks, communities can also expand, shrink, or dissolve. As argued by [73], communities in social media, in particular blogspace, are different from conventional web communities. "Within a community of interacting bloggers, a given topic may become the subject of intense debate for a period of time, then fade away. These bursts of activity are typified by heightened hyperlinking amongst the blogs involved — within a time interval." It entails a new task to discover communities behind noisy network interactions across time.

Essentially, social media networks are highly heterogeneous. It is often more than just a single social network. Heterogeneous types of interactions, entities are involved, and temporal evolution also comes into play. Note that these different types of networks are not exclusive. In reality, it is very likely that a network is multi-dimensional, multi-modal and dynamic simultaneously. Facing evolving networks with heterogeneous entities or interactions, communities in one dimension, one mode or one snapshot becomes correlated to other dimensions, mode or snapshots. Existent approaches for unsupervised learning on graphs have to be extended to handle new forms of networks.

3.2 Motivation

Social media offers an easily-accessible platform for diverse online social activities, but also introduces heterogeneity in networks. Thus, it calls for solutions to extract communities in heterogeneous networks. However, it remains unanswered why one cannot reduce a heterogeneous network to several homogeneous ones (i.e., only one dimension, one mode or one snapshot) for investigation.

The reason is that the interaction information in one mode or one dimension might be too noisy to detect meaningful communities. For instance, in the YouTube example in Figure 3.2. It seems acceptable if we only consider the user mode. In other words, just study the friendship network. On the one hand, some users might not have any online friends either because they are too introvert to talk to other online users, or because they just join the network and are not ready for or not interested in connections. On the other hand, some users might abuse connections, since

it is relatively easy to make connections in social media compared with in the physical world. As mentioned in [121], a user in Flickr can have thousands of friends. This can hardly be true in the real world. It might be the case that two online users get connected but they never talk to each other. Thus, these online connections of one mode or one dimension can hardly paint a true picture of what is happening. A single type of interaction provides limited (often sparse) information about the community membership of online users. Fortunately, social media provides more than just a single friendship network. A user might engage in other forms of activities besides connecting to friends. It is helpful to utilize information from other modes or dimensions for more effective community detection.

As for identifying community evolution in a dynamic network, one straightforward approach is to apply community detection to each snapshot of the network Such a scheme is adopted in [59, 73, 107, 7]. However, most community detection methods, such as classical k-means clustering algorithm, might return a local optimal. The initialization, the processing order of nodes, a slight change of network connection might lead to a dramatically different community structure result. For example, Hopcroft et al. [58] show that the community structure based on hierarchical agglomerative clustering can change sharply with the removal of one single edge in the network. Some communities are essentially random. Thus, for a community detection method that is unstable or have multiple local optimal solutions, it is difficult to conclude whether there is a community evolution between consecutive timestamps. Perhaps, it is simply the randomization due to the community detection algorithm being used. However, it is noticed that a network tend to evolve smoothly [28]. Rather than conducting community detection independently at each network snapshot, we may obtain more a robust community structure if temporal smoothness is employed smartly.

In sum, by integrating network information of multiple dimensions, modes or snapshots, we may be able to extract more robust communities, thus achieving accurate unsupervised learning. In the previous chapter, we proposed the concept of social dimension. By extracting social dimensions from a network, we convert a network into conventional attribute format for nodes. Below, we discuss strategies for integration of different types of networks following the social-dimension learning framework.

3.3 Social Dimension Integration for Unsupervised Learning

As mentioned in the previous section, social media networks can be multi-dimensional, multi-modal and dynamic. Communities of one dimension, one mode or one snapshot become correlated to other dimensions, mode or snapshots. We need to integrate heterogeneous information together in order to identify a more accurate community structure. For presentation convenience, we abbreviate *dimension, mode or snapshot* as DMS.

In order to find communities in one target DMS, we can follow a similar procedure as the SocioDim framework proposed in the previous chapter. The unsupervised learning for a target DMS can also be decomposed into two phases:

- Phase I: Extraction of social dimensions from related DMSs;
- Phase II: Integration of social dimensions via Spectral Analysis for target DMS.

As introduced in the previous chapter, by extracting social dimensions from a network, we convert the network into conventional attribute format, and the social dimensions can be considered as features of nodes in the network. Similarly, as for unsupervised learning, we also extract social dimensions from related DMSs. By integrating these related features, we can update the social dimension of one DMS via spectral analysis. In particular, we compute the singular value decomposition (SVD) of the integrated social dimensions. This process is quite similar to the latent semantic indexing (LSI) [12] commonly applied in text mining. Essentially, we extract the principal interaction patterns by integrating interaction information from related dimensions, modes or snapshots. Here, the unsupervised learning procedure differs from the supervised learning framework in the second phase. For supervised learning, we build a classifier based on social dimensions. Unsupervised learning, nevertheless, extracts communities by integrating related social dimensions. As the above procedure is just for one target DMS, we may iteratively update social dimensions for each DMS. Thus, we have a general framework for unsupervised learning in Figure 3.3. In each iteration, we extract social dimensions from related DMSs and conduct integration.

Here, we present a general unsupervised learning approach to handle various kinds of social media networks. So far, the proposed unsupervised learning approach seems quite abstract and ad

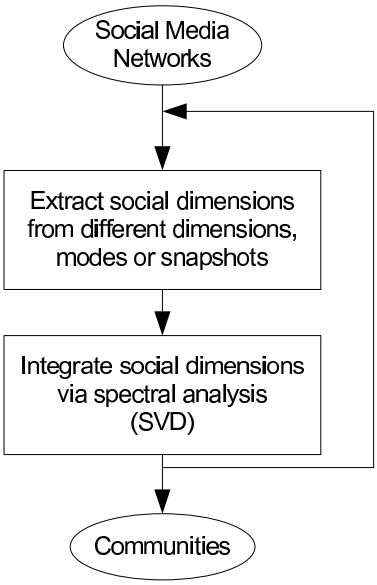


Figure 3.3: SocioDim Integration for Unsupervised Learning

hoc. Indeed, this framework can be instantiated depending on the network being studied. Next, we show how we instantiate this framework to find communities in different types of networks and what the reasoning is behind such a procedure. We will first study communities in multi-dimensional networks and then generalizes to dynamic multi-modal networks as well.

3.4 Communities in Multi-Dimensional Networks

A d-dimensional network is represented as

$$\mathscr{A} = \{A^{(1)}, A^{(2)}, \cdots, A^{(d)}\}$$

with $A^{(i)}$ represents the interaction among actors in the i-th dimension satisfying

$$A^{(i)} \in \mathbb{R}_{+}^{n \times n}, \;\; A^{(i)} = (A^{(i)})^{T}, \;\; i = 1, 2, \cdots, d$$

where n is the total number of actors involved in the network. Here, we concentrate on symmetric networks. Asymmetric networks can be converted into symmetric networks through certain operations as shown later. In a multi-dimensional network, actors interact with each other through multiple dimensions. As one user might not engage in all kinds of activities, we hope to find out the latent community structure among actors by integrating all types of network interactions.

3.4.1 Instantiation of SocioDim Integration

Social dimensions capture the diverse relations involved in a network. They can also be considered as structural features of nodes extracted from a network. Following the general framework in Figure 3.3, we can perform integration over social dimensions extracted from each type of interaction. One might conjecture that we can simply take the average of social dimensions as follows:

$$\bar{S} = \frac{1}{d}\sum_{i=1}^{d} S^{(i)} \tag{3.1}$$

where $S^{(i)} \in \mathbb{R}^{n\times\ell}$ denotes the social dimensions extracted from the i-th dimension of the network $A^{(i)}$. and d is the number of different types of interaction. Unfortunately, this straightforward extension does not apply to social dimensions. Because the social dimensions obtained through representative community detection methods such as latent space models, block models, spectral clustering or modularity maximization as discussed in Appendix A, are not unique. The social dimensions $S^{(i)}$ correspond to the top eigenvector of a utility matrix (see Eq. (A.14)). In the simplest case, if $S^{(i)}$ represents the top eigenvectors, then $-S^{(i)}$ is also a valid solution. Consequently, dissimilar social dimensions do not suggest that drastically different community structures.

Alternatively, we expect $S^{(i)}$ of different type of interaction to be highly correlated *after certain transformations*. To capture the correlations between multiple sets of variables, (generalized) canonical correlation analysis (CCA) [62, 69] is the standard statistical technique. CCA attempts to find a transformation for each set of variables such that the pairwise correlations are maximized. Here we briefly illustrate one scheme of generalized CCA which turns out to reduce to SVD in our specific case.

Let $S^{(i)} \in \mathbb{R}^{n\times\ell}$ denote the social dimensions extracted from the i-th dimension of the network, and $\mathbf{w_i} \in \mathbb{R}^{\ell}$ be the linear transformation applied to $S^{(i)}$. The correlation between two sets of social dimensions ($S^{(i)}$ and $S^{(j)}$) after transformation is

$$R(i,j) = (S^{(i)}\mathbf{w_i})^T(S^{(j)}\mathbf{w_j}) = \mathbf{w_i}^T\left((S^{(i)})^T S^{(j)}\right)\mathbf{w_j} = \mathbf{w_i}^T C_{ij}\mathbf{w_j}$$

with $C_{ij} = (S^{(i)})^T S^{(j)}$ representing the covariance between the social dimensions of the i-th and the j-th dimensions. Generalized CCA attempts to maximize the summation of pairwise correlations

as in the following form:

$$\max \quad \sum_{i=1}^{d}\sum_{j=1}^{d} \mathbf{w_i}^T C_{ij} \mathbf{w_j} \tag{3.2}$$

$$s.t. \quad \sum_{i=1}^{d} \mathbf{w_i}^T C_{ii} \mathbf{w_i} = 1 \tag{3.3}$$

Using standard Lagrange multiplier and setting the derivatives respect to $\mathbf{w_i}$ to zero, we obtain the equation below:

$$\begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1d} \\ C_{21} & C_{22} & \cdots & C_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ C_{d1} & C_{d2} & \cdots & C_{dd} \end{bmatrix} \begin{bmatrix} \mathbf{w_1} \\ \mathbf{w_2} \\ \vdots \\ \mathbf{w_d} \end{bmatrix} = \lambda \begin{bmatrix} C_{11} & 0 & \cdots & 0 \\ 0 & C_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{dd} \end{bmatrix} \begin{bmatrix} \mathbf{w_1} \\ \mathbf{w_2} \\ \vdots \\ \mathbf{w_d} \end{bmatrix} \tag{3.4}$$

Recall that our social dimensions extracted from each dimension is essentially the top eigenvectors of a utility matrix. They satisfy the orthogonal constraint, $(S^{(i)})^T S^{(i)} = I$. Thus, matrix $diag(C_{11}, C_{22}, \cdots, C_{dd})$ in Eq. (3.4) becomes an identity matrix. Therefore $\mathbf{w} = [\mathbf{w_1}^T, \mathbf{w_2}^T, \cdots, \mathbf{w_d}^T]^T$ corresponds to the top eigenvector of the full covariance matrix on the left side of Eq. (3.4), which is equivalent to PCA applied to data of the following form:

$$X = \left[S^{(1)}, S^{(2)}, \cdots, S^{(d)}\right] \tag{3.5}$$

Suppose the SVD of X is $X = U\Sigma V^T$, then $\mathbf{w}$ corresponds to the first column of V. Thus we have

$$\frac{1}{d}\sum_{i=1}^{d} S^{(i)} \mathbf{w_i} = \frac{1}{d}\left[S^{(1)}, S^{(2)}, \cdots, S^{(d)}\right] \mathbf{w} = \frac{1}{d} X V_1 = \frac{\sigma_1}{d} U_1$$

Since σ_1/d is a scalar, U_1 is essentially the average feature values of each actor after we aggregate the structural features of different dimensions along with the transformation $\mathbf{w}$. There are $k-1$ degrees of freedom with k communities. To compute the $(k-1)$-dimension embedding, we just need to project the data X onto the top $(k-1)$ principal vectors. It follows that the top $(k-1)$ vectors of U are the aggregated structural features.

The detailed social dimension integration algorithm is summarized in Figure 2.4. We first extract social dimensions from each type of interaction of the network via representative community detection methods; then PCA is applied on the concatenated data as in Eq. (3.5) to select the top

Algorithm: Unsupervised Learning based on Social Dimensions

Input: $Net = \{A^{(1)}, A^{(2)}, \cdots, A^{(d)}\}$,
number of communities k,
number of social dimensions to extract ℓ;
Output: community assignment *idx* for each node.

1. Extract ℓ social dimensions $S^{(i)}$ for each $A^{(i)}$;
2. Compute slim SVD of $X = [S^{(1)}, S^{(2)}, \cdots S^{(d)}] = UDV^T$;
3. Obtain lower-dimensional embedding $\widetilde{U} = U(:, k-1)$;
4. Normalize the rows of $\widetilde{U}$ to unit length;
5. Calculate the cluster *idx* with k-means on $\widetilde{U}$.

Figure 3.4: Algorithm: Social Dimension Integration for Multi-Dimensional Networks

eigenvectors. After projecting the data onto the principal vectors, we obtain a lower-dimensional embedding which captures the principal pattern across all the dimensions of the network. Then we can perform k-means on this embedding to find out the discrete community assignment.

Note that in the algorithm, we do not conduct iterative process as suggested in the framework in Figure 3.3. This is because we aim to find out the shared community structure across different types of interactions, rather than the community structure in each type of interaction. Next, we will compare the proposed social dimension integration with other approaches to deal with multi-dimensional networks.

3.4.2 Experiment Setup

In this part, we introduce the data set and baseline methods for comparison. As the data does not have the ground truth information about communities, we develop a novel evaluation strategy *cross-dimension network validation* following typical cross validation.

3.4.2.1 YouTube Data

YouTube[1] is currently the most popular video sharing web site. It is reported to "attract 100 million video views per day"[2]. As of March 17th, 2008, there had been 78.3 million videos uploaded, with over 200, 000 videos uploaded per day[3]. This media sharing site allows users to interact with each other in various forms such as contacts, subscriptions, sharing favorite videos, etc. We use YouTube

[1] http://www.youtube.com/
[2] http://www.usatoday.com/tech/news/2006-07-16-youtube-views_x.htm
[3] http://ksudigg.wetpaint.com/page/YouTube+Statistics?t=anon

Table 3.1: The Density of Each Dimension in the Constructed 5-Dimensional Network

Network	Dimension	Density
$A^{(1)}$	contact	6.74×10^{-4}
$A^{(2)}$	co-contact	1.71×10^{-2}
$A^{(3)}$	co-subscription	4.90×10^{-2}
$A^{(4)}$	co-subscribed	1.97×10^{-2}
$A^{(5)}$	favorite	3.34×10^{-2}

Data API to crawl the contacts network, subscription network as well as each user's favorite videos. We choose 100 authors who recently uploaded videos as the seed set for crawling, and expand the network via their contacts and subscriptions. We obtain a small portion of the whole network, with $30,522$ user profiles reaching in total $848,003$ contacts and $1,299,642$ favorite videos. After removing those users who decline to share their contact information, we have $15,088$ active user profiles as presented in three different interactions: two adjacency matrices of size $15,088 \times 848,003$ representing contact relationship, and subscriptions and a matrix of size $15,088 \times 1,299,642$ representing users' favorite videos.

One issue is that the collected subscription network is directional while most community detection methods such as block models, spectral clustering and modularity maximization, are proposed for undirected networks. For such cases, simply ignoring the direction can confuse the two roles of the directional interaction. Instead, we decompose the asymmetric interaction A into two unidirectional interactions:

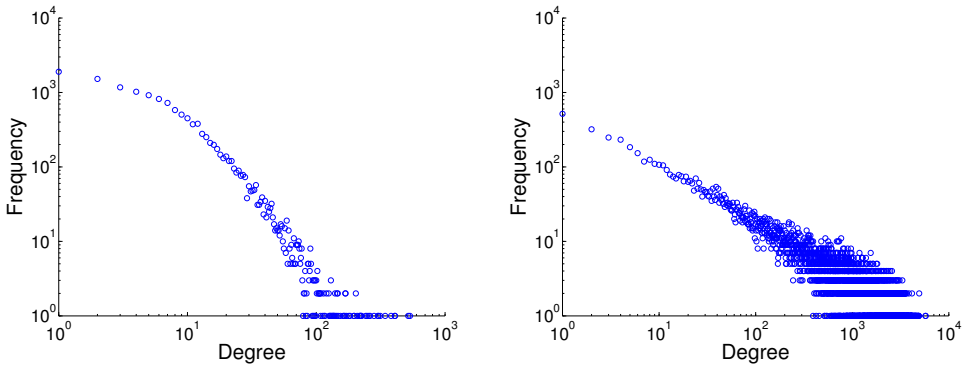
$$A' = AA^T; \tag{3.6}$$

$$A'' = A^T A. \tag{3.7}$$

Essentially, if two social actors both subscribe to the same set of users, it is likely that they are similar and share the same community; On the other hand, if two are referred by the same set of actors, their similarity tends to be higher than that of random pairs. This is similar to the two roles of hub and authority of web pages as mentioned in [70].

To utilize all aspects of information in our collected data, we construct a 5-dimensional network:

$A^{(1)}$: contact network: the contact network among those $15,088$ active users;



(a) Contact Dimension (b) Favorite Dimension

Figure 3.5: Power Law Distribution on Different Dimensions of Interaction

$A^{(2)}$: co-contact network: two active users are connected if they both add another user as contact; This is constructed based on all the reachable 848,003 users (excluding those active ones) in our collected data following Eq. (3.6).

$A^{(3)}$: co-subscription network: the connection between two users denotes they subscribe to the same user; constructed following Eq. (3.6);

$A^{(4)}$: co-subscribed network: two users are connected if they are both subscribed by the same user; constructed following Eq. (3.7);

$A^{(5)}$: favorite network: two users are connected if they share favorite videos.

All these different interactions are correlated with user interests. According to homophily effect well studied in social science [92], people tend to connect to others sharing certain similarities. Thus, we expect that connected friends in the contact network $A^{(1)}$ is more likely to share certain interests. Similarly, if both users connect to another user or a favorite video (as $A^{(2)}$, $A^{(3)}$ or $A^{(5)}$), they are likely to share certain interests. On the other hand, if two users are subscribed by the same set of users (as in $A^{(4)}$), their shared content, thus their interests, are similar. Essentially, we hope to extract communities share similar interests by integrating heterogeneous interactions.

Table 3.1 shows the connection density of each dimension. Contact dimension is the most sparse one, while the other dimensions, due to the construction, are denser. Figure 3.5 shows the degree distribution in contacts network and favorite network. Both, as expected, follow a power law [30].

3.4.2.2 Baseline Methods

We compare our social dimension integration scheme in Figure 3.4 with other baseline methods. Modularity maximization is used as the base community detection method to show the efficacy of our proposed approach. We will examine the following two questions:

- Is it necessary to integrate multiple types of interactions?
- If yes, which type of integration is more effective?

Thus, one baseline is to extract communities from only one type of interaction. Besides social dimension integration, other integration strategies following the general community detection process in Figure A.2 are:

- *Network Integration*. A simple strategy to handle a multi-dimensional network is to treat it as single-dimensional. One straightforward approach is to calculate the average interaction network among social actors:

$$\bar{A} = \frac{1}{d}\sum_{i=1}^{d} A^{(i)} \tag{3.8}$$

  Correspondingly,

$$\bar{m} = \frac{1}{d}\sum_{i=1}^{d} m^{(i)}, \;\; \bar{\mathbf{d}} = \frac{1}{d}\sum_{i=1}^{d} \mathbf{d}^{(i)} \tag{3.9}$$

  With $\bar{A}$, this boils down to classical community detection in a single-dimensional network. Based on the average network, we can follow the community detection process as stated in the unified view. Take modularity maximization as an example. we can maximize the modularity as follows:

$$\max_{S} \frac{1}{2\bar{m}} Tr\left(S^T\left[\bar{A} - \frac{\bar{\mathbf{d}}\bar{\mathbf{d}}^T}{2\bar{m}}\right]S\right) \tag{3.10}$$

- *Utility Integration*. Another variant for integration is to combine utility matrices instead of networks. We can obtain an average utility matrix as follows:

$$\bar{M} = \frac{1}{d}\sum_{i=1}^{d} M^{(i)}$$

where $M^{(i)}$ denotes the utility matrix constructed in the i-th dimension. The community indicators can be computed via the top eigenvectors of the utility matrix. This is equivalent to *optimizing the objective function over all the dimensions simultaneously*. As for modularity maximization, the average utility matrix in this case would be

$$\bar{M} = \frac{1}{d}\sum_{i=1}^{d}\widetilde{B}^{(i)} = \frac{1}{d}\sum_{i=1}^{d}\left\{\frac{A^{(i)}}{2m^{(i)}} - \frac{\mathbf{d}^{(i)}(\mathbf{d}^{(i)})^T}{(2m^{(i)})^2}\right\} \tag{3.11}$$

Finding out the top eigenvectors of the average utility matrix is equivalent to maximizing the average modularity as follows:

$$\max_S \frac{1}{d}\sum_{i=1}^{d} Tr(S^T\widetilde{B}^{(i)}S) = \max_S Tr(S^T\bar{M}S) \tag{3.12}$$

- *Partition Integration*. Partition integration takes effect after the community partition of each network dimension is ready. This problem has been studied as the *cluster ensemble* problem [115], which combines multiple clustering results of the same data from a variety of sources into a single consensus clustering. Strehl and Ghoph [115] propose three effective and comparable approaches: cluster-based similarity partitioning algorithm (CPSA), HypergGraph Partition Algorithm and Meta-Clustering Algorithm. For brevity, we only present the basic idea of CPSA here. CPSA constructs a similarity matrix from each clustering. Two objects' similarity is 1 if they belong to the same group, 0 if they belong to different groups. Let $H^{(i)} \in \{0,1\}^{n\times k}$ denote the community indicator matrix of clustering based on interactions at dimension i. The similarity between nodes can be computed as

$$\frac{1}{d}\sum_{i=1}^{d}H^{(i)}(H^{(i)})^T = \frac{1}{d}\sum_{i=1}^{d}\widetilde{H}\widetilde{H}^T \text{ where } \widetilde{H} = \left[H^{(1)}, H^{(2)}, \cdots, H^{(d)}\right]$$

Based on this similarity matrix between nodes, we can apply similarity-based community detection methods we introduced before to find out clusters. A disadvantage of this CPSA is that the computed similarity matrix can be dense, which might not be applicable to large networks. Instead, we can treat $\widetilde{H}$ as the feature representation of actors and cluster them based on k-means directly. Intuitively, if two actors are assigned to the same group in the majority of dimensions, they would share features. Thus, the two actors are likely to reside within the same community in the final consensus cluster as well.

Input: $Net = \{A^{(1)}, A^{(2)}, \cdots, A^{(d)}\}$;
a multi-dimensional integration scheme f;

Output: a community quality measure $Q^{(i)}$ for each network dimension.

1. **for** $p = 1, \ldots, d$
2. hold out $A^{(p)}$ for testing;
3. obtain community structure H by applying f to
training dimensions $\{A^{(1)}, \cdots, A^{(p-1)}, A^{(p+1)}, \cdots, A^{(d)}\}$;
4. compute the quality measure $Q^{(p)}$ based on H and $A^{(p)}$.
5. **end**

Figure 3.6: CDNV: Cross-Dimension Network Validation

3.4.2.3 Evaluation Strategy

We now discuss evaluation methods that are suitable for multi-dimensional networks. As we emphasized in the introduction of this dissertation, many social media networks do not provide ground truth information of communities. When ground truth is not available, an alternative evaluation method is needed to quantify community structures extracted employing different integration strategies. If a latent community structure is shared across network dimensions, we can perform *cross-dimension network validation* (CDNV) as in Figure 3.6. Given a multi-dimensional network $Net = \{A^{(i)} | 1 \leq i \leq d\}$, we can learn a community structure from $d-1$ dimensions of the network and check how well the structure matches the left-out dimension ($A^{(p)}$). In other words, we use $d-1$ dimensions for training and the remaining one for testing. During the training, we obtain some communities (C), and use C to calculate modularity for the data of $A^{(p)}$ as follows:

$$Q = \frac{1}{2m} \sum_{C} \sum_{i \in C, j \in C} A_{ij} - d_i d_j / 2m. \tag{3.13}$$

A larger modularity indicates a more accurate community structure discovered from training data.

3.4.3 Experiment Results

The four multi-dimensional integration schemes as well as community detection methods on a single dimension are compared. We cluster actors involved in the network into different numbers of communities. The clustering performance of single-dimensional and multi-dimensional methods when $k = 20$, 40 and 60 are presented in Table 3.2. In the table, $R^{(i)} (1 \leq i \leq 5)$ denotes the ranking of each method based on CDNV as using $A^{(i)}$ for testing, and $R_{average}$ the average ranking across

Table 3.2: Performance When Actors are Partitioned into 20, 40, and 60 Communities

k=20	Strategies	$R^{(1)}$	$R^{(2)}$	$R^{(3)}$	$R^{(4)}$	$R^{(5)}$	$R_{average}$
Single-Dimensional Community Detection	$A^{(1)}$	—	7	8	8	8	7.75
	$A^{(2)}$	4	—	5	5	6	5.00
	$A^{(3)}$	6	5	—	4	4	4.75
	$A^{(4)}$	7	4	4	—	5	5.00
	$A^{(5)}$	8	6	6	6	—	6.50
Multi-Dimensional Integration	Network	5	8	7	7	7	6.80
	Utility	2	2	2	2	2	2.00
	SocioDim	**1**	**1**	**1**	**1**	**1**	**1.00**
	Partition	3	3	3	3	3	3.00

k=40	Strategies	$R^{(1)}$	$R^{(2)}$	$R^{(3)}$	$R^{(4)}$	$R^{(5)}$	$R_{average}$
Single-Dimensional Community Detection	$A^{(1)}$	—	8	6	7	8	7.75
	$A^{(2)}$	4	—	4	5	6	4.75
	$A^{(3)}$	5	4	—	4	4	4.25
	$A^{(4)}$	7	6	5	—	7	6.25
	$A^{(5)}$	8	7	7	6	—	7.00
Multi-Dimensional Integration	Network	6	5	8	8	5	6.40
	Utility	2	2	2	3	2	2.20
	SocioDim	**1**	**1**	**1**	2	**1**	**1.20**
	Partition	3	3	3	**1**	3	2.60

k=60	Strategies	$R^{(1)}$	$R^{(2)}$	$R^{(3)}$	$R^{(4)}$	$R^{(5)}$	$R_{average}$
Single-Dimensional Community Detection	$A^{(1)}$	—	5	6	7	8	6.50
	$A^{(2)}$	3	—	5	4	6	4.50
	$A^{(3)}$	6	6	—	5	7	6.00
	$A^{(4)}$	7	4	4	—	5	5.00
	$A^{(5)}$	8	8	7	6	—	7.25
Multi-Dimensional Integration	Network	5	7	8	8	4	6.40
	Utility	2	2	2	**1**	2	1.80
	SocioDim	**1**	**1**	**1**	2	**1**	**1.20**
	Partition	4	3	3	3	3	3.20

all network dimensions. Bold entries denote the best in each column for each case. In the table, rows represent methods and columns denote the Note that in our cross-dimension network validation procedure, the test dimension is not available during training, thus the diagonal entries for single-dimensional methods are not shown.

SocioDim integration is clearly the winner most of the time, except for certain rare cases (e.g., using $A^{(4)}$ as the test dimension when $k = 40$ or 60). We notice that the rankings of different integration strategies do not change much with different k. A closer examination reveals that utilizing information of all the dimensions (except network integration) outperforms single-dimensional clustering. Network integration does not work well, because the network studied here are weighted and simple average blurs the latent community structure information presented in each dimension. In terms of performance ranking, SocioDim integration $\prec$ utility integration $\prec$ integration $\prec$ network integration, with SocioDim integration being the winner. SocioDim integration, by removing noise in each dimension, yields the most accurate community structure among all the methods.

Multi-dimensional networks commonly exist in many social networking sites, reflecting diverse user interactions. We formally describe the community detection problem in multi-dimensional networks and present one approach based on social dimensions to handle the problem. We show that SocioDim integration, which extracts social dimensions from each dimension of a multi-dimensional network and treat them as structural features to integrate them via singular value decomposition, outperforms other integration schemes. Next, we will show that the learning framework in Figure 3.3 can also be instantiated to handle dynamic multi-modal networks.

3.5 Generalization to Communities in Dynamic Multi-Modal Networks

Within a multi-modal network, different types of entities tend to form groups or communities. In the YouTube example in Figure 3.2, users sharing similar interests are more likely to form a group; videos are clustered naturally if they relate to similar contents; and tags are clustered if they are associated with similar users and videos. Generally, a user group may interact with multiple groups of another mode, i.e., users might have multiple interests, thus relating to various video groups.

In a multi-modal network, actors of different modes can evolve differently. For instance, in the previous Youtube example, the video clusters tend to be stable based on their semantic close-

ness. But users might divert their personal interest and join a different community for interaction. Facing heterogeneous entities with dynamic interactions, extracting evolving groups can lead to a clear understanding of interaction between disparate modes as well as long-term evolution patterns. This can benefit visualization of a complex network with heterogeneous entities and interactions, aid decision making in various domains, and signal an event alarm if undesirable evolution patterns are observed in the early stage. For example, to detect user interests shift for more effective targeted marketing, or to detect suspicious financial activities if an abnormal change in transactions is detected. However, the problem of discovering community evolution becomes challenging in dynamic multi-modal networks because 1) the evolutions of different modes become correlated; and 2) disparate modes demonstrate distinctive evolution patterns.

Below, we show that through a series of derivation, it reduces to the general framework presented in Figure 3.3 to find communities in a dynamic multi-modal network.

3.5.1 Problem Formulation

In order to extract communities in a multi-modal network, we follow the framework presented in [84]. With a little bit abuse, we use similar notations as in [84] which are different from the rest of this work. Given an m-mode network with m types of actors $\mathbb{X}_1, \mathbb{X}_2, \cdots, \mathbb{X}_m$, we aim to find how the latent community of each mode evolves. In our framework, we only consider discrete timestamps by taking a series of snapshots, which is commonly adopted in network analysis with temporal information [9, 78]. For a snapshot at time t, a network N^t is represented as multiple interactions between different modes. Let $R^t_{i,j} \in \mathbb{R}^{n_i \times n_j}$ denote the interaction between two modes $\mathbb{X}_i$ and $\mathbb{X}_j$ at timestamp t, n_i and n_j the number of actors at mode i and j, k_i and k_j the number of latent communities for $\mathbb{X}_i$ and $\mathbb{X}_j$ respectively.

In order to extract communities at each mode, we assume interactions between modes can be approximated by interactions between groups [85, 34]. Thus we can approximate the original network through interaction blocks. In particular,

$$R^t_{i,j} \approx C^{(i,t)} A^t_{i,j} (C^{(j,t)})^T$$

where $C^{(i,t)} \in \{0,1\}^{n_i \times k_i}$ denotes latent cluster (block) membership for $\mathbb{X}_i$ at timestamp t, and $A_{i,j}$ represents the density of group (block) interaction. In other words, the group membership de-

termines how two actors interact. This essentially hinges on a similar assumption as stochastic block models [105]. The difference is that stochastic block models deal with the problem from a probabilistic aspect. Here we identify the block structure of multi-modal networks via matrix approximation:

$$\min \quad ||R^t_{i,j} - C^{(i,t)} A^t_{i,j} (C^{(j,t)})^T||^2_F \tag{3.14}$$

$$s.t. \quad C^{(i,t)} \in \{0,1\}^{n_i \times k_i}, \ \sum_{k=1}^{k_i} C^{(i,t)}_{*k} = 1 \tag{3.15}$$

$$C^{(j,t)} \in \{0,1\}^{n_j \times k_j}, \ \sum_{k=1}^{k_j} C^{(j,t)}_{*k} = 1 \tag{3.16}$$

The constraints in (3.15) and (3.16) force each row of the indicator matrix to have only one entry being 1. That is, each actor belongs to only one community. Unfortunately, the discreteness of the constraints makes the problem NP-hard. A strategy that has been well studied in spectral clustering [157] is to allow the cluster indicator matrix to be continuous and relax the hard clustering constraint as follows:

$$(C^{(i,t)})^T C^{(i,t)} = I_{k_i}$$

In a multi-modal network, diverse interactions occur between different modes. Hence, the objective in Eq. (3.14) can be changed to

$$\min \sum_{1 \le i < j \le m} w_a^{(i,j)} ||R^t_{i,j} - C^{(i,t)} A^t_{i,j} (C^{(j,t)})^T||^2_F \tag{3.17}$$

with $w_a^{(i,j)}$ being the weights associated with different interactions.

With a dynamic multi-modal network, we have multiple snapshots of the network. Naturally, the objective function *without* considering its temporal effect can be formulated as $\mathbf{F_1}$:

$$\min \sum_{t=1}^{\mathbb{T}} \sum_{1 \le i < j \le m} w_a^{(i,j)} ||R^t_{i,j} - C^{(i,t)} A^t_{i,j} (C^{(j,t)})^T||^2_F \tag{3.18}$$

$$s.t. (C^{(i,t)})^T C^{(i,t)} = I_{k_i} \quad i = 1, \cdots, m, \ \ t = 1, \cdots \mathbb{T} \tag{3.19}$$

Then we have the following theorem:

Table 3.3: Symbols and Denotations

Symbol	Denotation
m	number of modes
n_i	number of actors at mode i
$\mathbb{X}_i$	entities at mode i
$R^t_{i,j}$	the interaction between two modes i and j at time t
k_i	number of latent groups of mode i
$C^{(i,t)}$	the community indicator matrix of mode i at time t
$A^t_{i,j}$	group interaction density between modes i and j
$w_a^{(i,j)}$	weight associated with interaction between modes i and j
$w_b^{(i)}$	weight associated with temporal regularization

Theorem 1 *Let $C^{(i,t)}, 1 \le i \le m, 1 \le t \le \mathbb{T}$ be a valid solution of $\mathbf{F_1}$, then $\widetilde{C}^{(i,t)}$ defined below is also a valid solution with the same objective value.*

$$\begin{aligned} & \widetilde{C}^{(i,t)} = C^{(i,t)} Q^{(i,t)} \\ s.t. \quad & (Q^{(i,t)})^T Q^{(i,t)} = Q^{(i,t)} (Q^{(i,t)})^T = I_{k_i} \\ & Q^{(i,t)} \in \mathbb{R}^{k_i \times k_i} \end{aligned}$$

Proof It suffices to show that the value of each single term in $\mathbf{F_1}$ does not change. Given a solution $C^{(i,t)}$ and $A^t_{i,j}$, we can choose $\widetilde{A}^t_{i,j} = (Q^{(i,t)})^T A^t_{i,j} Q^{(j,t)}$, then

$$\widetilde{C}^{(i,t)} \widetilde{A}^t_{i,j} (\widetilde{C}^{(j,t)})^T = C^{(i,t)} A^t_{i,j} (C^{(j,t)})^T$$

which completes the proof. ∎

The formulation $\mathbf{F_1}$ does not consider the relationship between consecutive timestamps. Solving $\mathbf{F_1}$ boils down to perform clustering at each snapshot independently. In reality, communities tend to evolve gradually. To obtain smooth community evolution, we add a temporal regularization term Ω which forces the clustering sequence to be smooth across different timestamps:

$$\Omega = \frac{1}{2} \sum_{t=2}^{\mathbb{T}} ||C^{(i,t)} (C^{(i,t)})^T - C^{(i,t-1)} (C^{(i,t-1)})^T ||_F^2 \tag{3.20}$$

Here, the coefficient $1/2$ is included due to notational conveniences for later derivation. Indeed, we are making a first-order Markov assumption. That is, the current clustering should be similar to the

clustering at the previous timestamp. Note that we do not take the regularization as

$$\Omega = \sum_{t=2}^{\mathbb{T}} ||C^{(i,t)} - C^{(i,t-1)}||_F^2 \tag{3.21}$$

which seems more natural at first glimpse. As demonstrated in Theorem 1, $C^{(i,t)}$ is equivalent under an orthogonal transformation. Hence, comparing $C^{(i,t)}$ and $C^{(i,t-1)}$ directly as in Eq. (3.21) does not necessarily capture the difference between the cluster indicators at different timestamps. On the contrary, the regularization term of Eq.(3.20) is independent of the orthogonal transformation, thus captures the difference of community structure of neighboring timestamps. With this regularization, the problem of identifying evolving groups can be formulated as:

$$\begin{aligned}
&\mathbf{F_2}: \min \sum_{t=1}^{\mathbb{T}} \sum_{i<j} w_a^{(i,j)} ||R_{i,j}^t - C^{(i,t)} A_{i,j}^t (C^{(j,t)})^T||_F^2 + \\
&\frac{1}{2} \sum_{i=1}^{m} w_b^{(i)} \sum_{t=2}^{\mathbb{T}} ||C^{(i,t)}(C^{(i,t)})^T - C^{(i,t-1)}(C^{(i,t-1)})^T||_F^2 \qquad (3.22)\\
&s.t.\ (C^{(i,t)})^T C^{(i,t)} = I_{k_i} \quad i = 1, \cdots, m,\ t = 1, \cdots \mathbb{T} \qquad (3.23)
\end{aligned}$$

with $w_b^{(i)}$ being the trade-off between the block model approximation of interactions and the temporal regularization. As evolution takes effect gradually, we aim to find a community structure that is consistent with current interaction matrix, whereas not drastically different from that of the previous timestamp.

3.5.2 Theoretical Derivation

To capture evolving groups in dynamic multi-modal networks, we have to solve $\mathbf{F_2}$. There is no analytical solution to the problem, but an iterative algorithm can be derived. We show that a closed-form solution exists for $A_{i,j}^t$ and $C^{(i,t)}$ if other variables are fixed [125]. Then we present the algorithm in an attribute view for easy comprehension and extension.

Theorem 2 *Given $C^{(i,t)}$, the optimal group interaction matrix $A_{i,j}^t$ can be calculated as*

$$A_{i,j}^t = (C^{(i,t)})^T R_{i,j}^t C^{(j,t)}$$

Proof Since $A^t_{i,j}$ appears in only one term in $\mathbf{F_2}$, we can focus on the single term to optimize $A^t_{i,j}$.

$$\begin{aligned} & ||R^t_{i,j} - C^{(i,t)} A^t_{i,j} (C^{(j,t)})^T||^2_F \\ = \; & tr[(R^t_{i,j} - C^{(i,t)} A^t_{i,j} (C^{(j,t)})^T)(R^t_{i,j} - C^{(i,t)} A^t_{i,j} (C^{(j,t)})^T)^T] \\ = \; & tr[R^t_{i,j} (R^t_{i,j})^T - 2C^{(i,t)} A^t_{i,j} (C^{(j,t)})^T (R^t_{i,j})^T + A^t_{i,j} (A^t_{i,j})^T] \end{aligned}$$

The last equation follows as $tr(AB) = tr(BA)$ and $C^{(i,t)}$ and $C^{(j,t)}$ are column orthogonal as in Eq. (3.23). Taking the derivative of the last equation with respect to $A^t_{i,j}$ to zero, we have

$$A^t_{i,j} = (C^{(i,t)})^T R^t_{i,j} C^{(j,t)}.$$

∎

Given the optimal $A^t_{i,j}$, it can be verified that

$$||R^t_{i,j} - C^{(i,t)} A^t_{i,j} (C^{(j,t)})^T||^2_F = ||R^t_{i,j}||^2_F - ||(C^{(i,t)})^T R^t_{i,j} C^{(j,t)}||^2_F. \tag{3.24}$$

Meanwhile,

$$\begin{aligned} & \frac{1}{2} ||C^{(i,t)} (C^{(i,t)})^T - C^{(i,t-1)} (C^{(i,t-1)})^T||^2_F \\ = \; & \frac{1}{2} tr\left[C^{(i,t)} (C^{(i,t)})^T + C^{(i,t-1)} (C^{(i,t-1)})^T - 2C^{(i,t)} (C^{(i,t)})^T C^{(i,t-1)} (C^{(i,t-1)})^T \right] \\ = \; & k_i - ||(C^{(i,t)})^T C^{(i,t-1)}||^2_F. \end{aligned} \tag{3.25}$$

Since $||R^t_{i,j}||^2_F$ in (3.24) and k_i in (3.25) are constants, we can transform $\mathbf{F_2}$ into the following objective:

$$\begin{aligned} \mathbf{F_3}: \quad \max \quad & \sum_{t=1}^{\mathbb{T}} \sum_{l \le i < j \le m} w^{(i,j)}_a ||(C^{(i,t)})^T R^t_{i,j} C^{(j,t)}||^2_F \\ & + w^{(i)}_b \sum_{t=2}^{\mathbb{T}} \sum_{i=1}^{m} ||(C^{(i,t)})^T C^{(i,t-1)}||^2_F \end{aligned} \tag{3.26}$$

Note that $C^{(i,t)}$ is interrelated with both $C^{(j,t)}$ and $C^{(i,t-1)}$. In general, there is no analytical closed-form solution. But the optimal $C^{(i,t)}$ can be obtained directly if $C^{(j,t)}$ and $C^{(i,t\pm1)}$ are given, which is stated in the following theorem.

Theorem 3 *Given $C^{(j,t)}$ and $C^{(i,t\pm1)}$, $C^{(i,t)}$ can be computed as the top left singular vectors of the matrix P_i^t concatenated by the following matrices in column-wise:*

$$P_i^t = \left[\left\{ \sqrt{w_a^{(i,j)}} R_{i,j}^t C^{(j,t)} \right\}_{j \neq i}, \sqrt{w_b^{(i)}} C^{(i,t\pm1)} \right] \tag{3.27}$$

Proof We focus only on those terms involving $C^{(i,t)}$ in the objective function. Without loss of generality, we discuss the cases when $2 \leq t \leq \mathbb{T}-1$ first.

$$\begin{aligned} L &= \sum_{i<j} w_a^{(i,j)} ||(C^{(i,t)})^T R_{i,j}^t C^{(j,t)}||_F^2 + \sum_{k<i} w_a^{(k,i)} ||(C^{(k,i)})^T R_{k,i}^t C^{(i,t)}||_F^2 \\ &\quad + w_b^{(i)} ||(C^{(i,t)})^T C^{(i,t-1)}||_F^2 + w_b^{(i)} ||(C^{(i,t+1)})^T C^{(i,t)}||_F^2 \\ &= tr\left[(C^{(i,t)})^T M_i^t C^{(i,t)} \right] \end{aligned} \tag{3.28}$$

where M_i^t is defined as

$$\begin{aligned} M_i^t &= \sum_{i<j} w_a^{(i,j)} R_{i,j}^t C^{(j,t)} (C^{(j,t)})^T (R_{i,j}^t)^T + \sum_{k<i} w_a^{(k,i)} (R_{k,i}^t)^T C^{(k,t)} (C^{(k,t)})^T R_{k,i}^t \\ &\quad + w_b^{(i)} C^{(i,t-1)} (C^{(i,t-1)})^T + w_b^{(i)} C^{(i,t+1)} (C^{(i,t+1)})^T \end{aligned} \tag{3.29}$$

So the problem boils down to a max-trace problem with orthogonality constraint as in Eq. (3.19). According to Ky-Fan theorem [14], this max-trace problem has a closed-form solution, which corresponds to the subspace spanned by the top k_i eigenvectors of M_i^t.

Note that M_i is a square symmetric matrix of size $n_i \times n_i$. When the number of actors in a mode $\mathbb{X}_i$ is huge, direct calculating M_i^t and its eigenvectors can be problematic. Alternatively, we represent M_i^t in the following matrix form:

$$M_i^t = P_i^t * (P_i^t)^T \tag{3.30}$$

where P_i^t is concatenated by the following matrices in *column-wise* (the order of terms does not matter):

$$\begin{aligned} P_i^t &= \left[\left\{ \sqrt{w_a^{(i,j)}} R_{i,j}^t C^{(j,t)} \right\}_{i<j}, \left\{ \sqrt{w_a^{(k,i)}} (R_{k,i}^t)^T C^{(k,t)} \right\}_{k<i}, \sqrt{w_b^{(i)}} C^{(i,t-1)}, \sqrt{w_b^{(i)}} C^{(i,t+1)} \right] \\ &= \left[\left\{ \sqrt{w_a^{(i,j)}} R_{i,j}^t C^{(j,t)} \right\}_{j \neq i}, \sqrt{w_b^{(i)}} C^{(i,t\pm1)} \right]. \end{aligned}$$

Input: R, k_i, $w_a^{(i,j)}$, $w_b^{(i)}$;
Output: $idx^{(i,t)}$, $C^{(i,t)}$, $A_{i,j}^t$.

1. Generate initial cluster indicator matrix $C^{(i,t)}$.
2. **Repeat**
3. **For** $t = 1 : \mathbb{T}$, $i = 1 : m$
4. shrink / expand $C^{(i,t\pm1)}$ if necessary;
5. calculate P_i^t (or M_i^t) as in Theorem 3;
6. calculate SVD of P_i^t (or eigenvectors of M_i^t);
7. update $C^{(i,t)}$ as top left singular (eigen) vectors;
8. **Until** the relative change of the objective (**F**3) $\leq \varepsilon$.
9. calculate $A_{i,j}^t$ as in Theorem 2;
10. calculate the cluster $idx^{(i,t)}$ with k-means on $C^{(i,t)}$.

Figure 3.7: SocioDim Integration for Dynamic Multi-Modal Networks

For $t = 1$ or $t = \mathbb{T}$, we only need to keep $C^{(i,t+1)}$ or $C^{(i,t-1)}$ respectively, instead of $C^{(i,t\pm1)}$. Typically the size of P_i^t is much smaller compared to M_i^t if the number of clusters of each mode is small. Let the SVD of P_i as $P_i^t = U\Sigma V$, then $M_i^t = U\Sigma^2 U^T$. That is, the top left-singular vectors of P_i^t correspond to the top eigenvectors of M_i^t, which completes the proof. ∎

3.5.3 Instantiation of SocioDim Integration

To solve the problem $\mathbf{F_3}$ in Eq. (3.26), we resort to alternating optimization. That is, we fix all the other variables while solving $C^{(i,t)}$. This process is iterated until convergence[4]. After convergence, $\{C^{(i,t)}\}$ are the approximate community indicator matrices. One problem to be addressed is to recover the discrete partition of communities. One commonly used post-processing scheme is to apply k-means clustering to the community indicators [8]. Combined with the results of previous sections, we have the algorithm in Figure 2.4.

Note that this is exactly an instantiation of the unsupervised learning framework in Figure 3.3. Each step to update $C^{(i,t)}$ corresponds to the left singular vectors of P_i^t, which is defined as in Eq. (3.27). P_i^t is constructed based on social dimensions from related modes and timestamps. The related social dimensions are concatenated together to form an "instance-attribute" matrix for nodes at mode i. By performing SVD, we obtain the corresponding social dimension C_i^t in mode i.

[4]The convergence property is discussed in detail in Appendix B.

Based on the proposed SocioDim integration algorithm, it is simple to extend our framework to handle networks of various properties.

Within-Mode Interactions. A social media network, for instance, can be both multi-modal and multi-dimensional. One can combine the two algorithms to handle multi-modal and multi-dimensional challenges. The combination is straightforward: if there are within-mode interactions that are multi-dimensional, we can simply append to P_i in Eq. (3.27) with social dimensions from each type of interaction. That is,

$$P_i = \left[\left\{ \sqrt{w_{ij}}\, R_{i,j} C_j \right\}_{i<j},\ \ \left\{ \sqrt{w_{ki}}\, R_{k,i}^T C_k \right\}_{k<i},\ \ \{C_i^d\} \right] \tag{3.31}$$

where C_i^d denotes the structural features extracted from dth dimension of interaction in the ith mode. In this way the presented unsupervised learning algorithm is able to handle diverse evolving heterogeneous networks in social media.

Actor Attributes. The attribute view of the algorithm allows for the integration of entity attributes in a convenient way. When a multi-modal network has attributes for certain modes of actors, we can simply add these attributes as features in P_i^t (Eq. (3.27)) when updating the cluster indicator matrix. That is:

$$\widetilde{P}_i^t = \left[\left\{ \sqrt{w_a^{(i,j)}} R_{i,j}^t C^{(j,t)} \right\}_{j \neq i}, \sqrt{w_b^{(i)}} C^{(i,t\pm 1)}, \sqrt{w_c^{(i)}} F_i^t \right]$$

where F_i^t denotes the attributes for actors in mode i, and $w_c^{(i)}$ the weight for actor attributes.

Higher-Order Temporal Regularization. In our formulation, we make the first-order Markov assumption that a community snapshot should be similar to its previous snapshot. In some domains, it might be necessary to include higher-order temporal regularization. An ℓ-th order temporal regularization enforces the community structure at timestamp t to be similar to that of $t-1$, $t-2$, $\cdots$, $t-\ell$. Based on the attribute view of the algorithm, we just need to modify P_i^t as follows to reflect the higher-order dependency:

$$\left[\left\{ \sqrt{w_a^{(i,j)}} R_{i,j}^t C^{(j,t)} \right\}_{j \neq i}, \left\{ \sqrt{w_b^{(i)} \alpha^{k-1}} C^{(i,t\pm k)} \right\}_{k=1,\cdots,\ell} \right]$$

Here, $0 < \alpha \leq 1$ is a decay factor to tune the regularization effect of different snapshots. Those snapshots that are close to current timestamp play a more important role in regularization.

Dormant and Emerging Actors. Some actors might become inactive at a certain time period. Meanwhile, new actors might join the network at a specific timestamp. The framework earlier assumes actors do not change. Here, we analyze more realistic cases where actors become dormant or join a network.

Note that our derived algorithm just uses $C^{(i,t\pm1)}$ as attributes for current timestamp. When an actor leaves or hibernates at certain timestamp, we can delete his corresponding entry in $C^{(i,t\pm1)}$ when updating $C^{(i,t)}$. However, after deletion, the orthogonality constraints might not be satisfied. Since our framework is an approximation to hard clustering, a tiny deviation from orthogonality does not affect the performance. However, when many actors become inactive, the total weights of remaining actors in $C^{(i,t\pm1)}$ is relatively small, thus playing a less important role in the computation of the similarity matrix M_i^t in Eq. (3.29). This is reasonable. A drastic membership change denotes the latent community is experiencing an "overhaul", thus it is not necessary to over-regularize the temporal change. When mode $\mathbb{X}_i$ at timestamp t has new actors, we simply set their entries in $C^{(i,t\pm1)}$ to 0.

Online Clustering. In some real-world scenarios, one might need to track the group evolution in an online fashion, and the algorithm should be modified accordingly. Instead of iteratively updating $C^{(i,t)}$, we update the community structure at different timestamps only once. Moreover, the corresponding updating matrix P_i^t and M_i^t involve only fixed $C^{(i,t-1)}$, but not $C^{(i,t+1)}$. That is,

$$\widetilde{P_i^t} = \left[\left\{ \sqrt{w_a^{(i,j)}} R_{i,j}^t C^{(j,t)} \right\}_{j\neq i}, \sqrt{w_b^{(i)}} C^{(i,t-1)} \right].$$

In a nutshell, actor attributes, social dimensions at different snapshots and interactions provide additional information for community detection. Based on the SocioDim integration framework, it is easy to extend the algorithm 3.7 to handle various cases. Such a unsupervised learning approach has been shown to outperform those without integration [125].

3.6 Related Work

As we presented unsupervised learning tasks concerning various types of social media networks, we discuss the related work for each, respectively.

3.6.1 Community Detection with Multiple Networks

Some work attempts to address unsupervised learning with multiple data sources or clustering results, such as *cluster ensemble* [115, 139, 39] and *consensus clustering* [95, 63, 103, 49]. These methods essentially fall into partition integration scheme presented in our framework. Most of the algorithms aim to find a robust clustering based on multiple clustering results, which are prepared via feature or instance sampling or disparate clustering algorithms. A similar idea is applied to community detection in social networks [58]. A small portion of connections between nodes are randomly removed before each run, leading to multiple different clustering results. Those clusters occurring repeatedly are considered more stable, and are deemed to reflect the natural communities in reality. However, all the cluster ensemble methods concentrate on either attribute-based data or one-dimensional networks.

Another related field is multi-view clustering. Bickel and Scheffere [15] propose co-EM and an extension of k-means and hierarchical clustering to handle data with two conditional independent views. Sa [31] creates a bipartite based on the two views and tries to minimize the disagreement. Different spectral frameworks with multiple views are studied in [159] and [83]. The former defines a weighted mixture of random walk over each view to identify communities. The latter assumes clustering membership of each view is provided and finds an optimal community pattern via minimizing the divergence of the transformed optimal pattern and the community membership of each view. A variant of utility integration based on block model approximation plus regularization is presented in [132]. Similarly, [6] suggests combining graph Laplacians for semi-supervised learning. It is empirically verified that our proposed integration schemes also apply to spectral clustering and block model approximation, and feature integration tends to be the most robust one.

Unsupervised multiple kernel learning [144] is relevant to network integration if we deem each dimension of the network as a similarity or kernel matrix. Multiple kernel learning [119] aims to find a combination of kernels to optimize for classification or clustering. Unfortunately, its limited scalability hinders its application even to a medium-size network.

3.6.2 Community Detection in Multi-Modal Networks

An extensive body of work studies the structural property of interactions between actors. One probabilistic approach is the stochastic block model [105], in which the link between actors is generated conditioned on the latent cluster membership of actors. Two actors within the same cluster are *stochastically equivalent*. That is, the interactions between (A_1, B_1) and (A_2, B_2) have the same probability if A_1 and A_2, B_1 and B_2 belong to the same cluster, respectively. Long et al. [86] propose a similar probabilistic framework to handle multi-modal networks with interactions and actor attributes. Topic models [16] are also extended to model documents within a social network [160, 146, 91]. Typically, they are specific for one type of document like Emails or papers.

Another attempt to model the structure is the latent space model. Intuitively, latent space models map actors to a latent low-dimensional space such that the actors whose positions are closer are more likely to interact with each other [57, 52]. Globerson et al. [48] study a two-mode network (authors and words) and map both authors and words into the same Euclidean space.

Spectral relational clustering, relevant to multi-modal networks, tries to discover latent community structures based on multiple relational tables. As the original problem of finding discrete cluster assignment (e.g., the entries of membership vector are either 0 or 1) is NP-hard, spectral clustering relaxes the constraint to allow the membership vector to be continuous. The initial work of co-clustering [33, 157, 85] tries to address the problem of clustering both words and documents simultaneously by taking advantage of the structure of a bipartite. Gao et al. [45] extend the problem to a star-typed network with multiple heterogeneous objects, and propose semi-definite programming to solve the problem. Alternatively, reinforcement clustering is proposed [145]. Long et al. [84] present a general spectral clustering framework to handle multi-type relational clustering with different kinds of objects and attributes, and an alternating optimization algorithm is presented to find a solution.

3.6.3 Community Evolution in Dynamic Networks

Temporal change of social networks has been attracting increasing attention [20, 107, 7]. It is empirically observed that some real-world networks are evolving [72]. Practitioners try to investigate how a network evolves and what might be a reasonable generative process to model the dynamics [78].

The critical factors to determine the group evolution are also reported [9].

On the other hand, evolutionary clustering [24] is developed. It assumes clustering result of current situation is similar to that of previous timestamps. Given multiple snapshots of network data, evolutionary clustering finds out a sequence of clustering with temporal smoothness [28, 81]. A Bayesian interpretation is presented in [154]. The latent space model with temporal change is also developed [110]. Tantipathananandh et al.[134] propose a general framework to handle dynamic single-mode network by casting it as a graph coloring problem and some greedy heuristics or approximation algorithms [133] are developed to handle large-scale data. Sun et al. [116] address the group evolution problem from an information-theoretic view. They present a scheme that detects not only the community structure but also the change point. As existing evolutionary approaches often require a specified number of clusters at each time stamp, (hierarchical) Dirichlet process is employed so that this parameter can be learned automatically from data [151, 150]. Most of the aforementioned works focus on data with attributes or single-mode networks. In this work, we explore the community evolution in multi-modal networks and show that it reduces to our proposed SocioDim integration framework for unsupervised learning.

3.7 Summary

Social media networks present many forms of heterogeneity. The interactions in social media can be multi-dimensional, multi-modal and dynamic with evolutions. As most interactions online are very noisy, it is imperative to integrate interaction information from multiple dimensions, modes or snapshots in order to infer the latent community structure among actors more accurately. In this chapter, we extend the SocioDim framework proposed in Chapter 2 and propose a SocioDim integration framework for unsupervised learning A key component in the SocioDim integration framework is to *extract social dimensions from related network dimensions, modes or snapshots, and then concatenate them all as conventional attributes for integration via spectral analysis*. To show the effect of the framework, we present two instantiations. One is to find shared community structure across multiple types of interactions in a multi-dimensional network and it is empirically shown to outperform community detection on a single type of interaction and other strategies of integration. The other is to identify evolving communities in a dynamic multi-modal network. In this

case, the SocioDim integration strategy indeed has a tight bond to block models and evolutionary clustering extended to dynamic multi-modal networks. Though the objective function and subsequent derivation might look a little bit twisted, the final algorithm reduces to SocioDim integration exactly. This explains why the proposed framework is sensible. With this unified SocioDim integration framework for unsupervised learning, it is easy to handle social media networks of various properties, and identify accurate latent community structures by integrating network information from multi-dimensional, multi-modal and dynamic interactions.

Chapter 4

SCALABLE LEARNING BASED ON SPARSE SOCIAL DIMENSIONS

In the previous two chapters, based on the concept of social dimension, we have presented a framework for learning with social media networks. The proposed learning framework is composed of two steps: 1) social dimension extraction, and 2) discriminative learning for supervised learning or spectral analysis for unsupervised learning. In the first step, latent social dimensions are extracted based on network topology to capture the potential affiliations of actors. These extracted social dimensions represent how each actor is involved in diverse relations. These social dimensions can be treated as features of actors for the subsequent learning. As one actor is likely to participate in multiple different relations, a soft clustering method is employed to extract social dimensions. However, as we show below, this soft clustering strategy has a serious limitation, i.e., it produces dense social dimensions, posing thorny challenges about scalability for both the extraction of social dimensions and the subsequent learning. In this chapter, following the proposed learning framework, we present techniques that are scalable to deal with networks of millions of nodes.

4.1 Problem Statement

In this chapter, we mainly focus on supervised learning. The derived techniques can be applied to unsupervised learning as well. Essentially, we focus on the same supervised learning problem in Chapter 2. For presentation convenience, we restate the problem again.

Given:

- K categories $\mathscr{Y} = \{\mathscr{Y}_1, \cdots, \mathscr{Y}_K\}$;
- a network $\mathscr{A} = (V, E, Y)$ representing the interactions between nodes, where V is the vertex set, E is the edge set, and each node v_i is associated with class labels $\mathbf{y}_i$ whose value can be unknown;
- the known labels Y^L for a subset of nodes V^L in the network, where $V^L \subseteq V$ and $y_{ij} \in \{+, -\}$ denotes the class label of the vertex v_i with respect to category $\mathscr{Y}_j$.

Find:

- the unknown labels Y^U for the remaining vertices $V^U = V - V^L$.

The emphasis in this chapter is about scalability. In particular, the given network $\mathscr{A}$ consists of millions of nodes that many methods might suffer from its size. We will analyze the bottleneck of existent methods and then propose alternative effective methods to address the scalability.

4.2 Motivation

In the initial instantiation of the learning framework *SocioDim* (see section 2.3), a spectral variant of modularity maximization [100] is adopted to extract social dimensions. The social dimensions correspond to the top eigenvectors of a modularity matrix. It has been empirically shown that this framework outperforms other representative relational learning methods on social media data. However, there are several concerns about the scalability of *SocioDim* with modularity maximization:

- Social dimensions extracted according to modularity maximization are dense. Suppose there are 1 million actors in a network and $1,000$ dimensionsare extracted. If standard double precision numbers are used, holding the full matrix alone requires $1M \times 1K \times 8 = 8G$ memory. This large-size dense matrix poses thorny challenges for the extraction of social dimensions as well as subsequent discriminative learning.

- Modularity maximization requires the computation of the top eigenvectors of a modularity matrix of the same size as a given network. When the network scales to millions of actors, the eigenvector computation becomes a daunting task.

- Networks in social media tend to evolve, with new members joining, and new connections occurring between existing members each day. This dynamic nature of networks entails efficient update of the model for collective behavior prediction. Efficient online update of eigenvectors with expanding matrices remains a challenge.

Such a limitation applies to other commonly used soft community detection approaches such as spectral clustering[88], block models[3] and probabilistic soft clustering[155]. Consequently, it is imperative to develop *scalable methods that are capable of handling large-scale networks efficiently without extensive memory requirement*.

4.3 Learning with Sparse Social Dimensions

Though SocioDim with soft clustering for social dimension extraction demonstrates promising results, its scalability is limited. A network could be sparse, the extracted social dimensions, however, are not sparse. Let's look at the toy network in Figure 4.1. Its social dimensions following modularity maximization are shown in Table 4.1. Clearly, none of the entries is zero. When a network expands into millions of actors, a reasonably large number of social dimensions need to be extracted. The corresponding memory requirement hinders both the extraction of social dimensions and the subsequent discriminative learning. Hence, it is imperative to develop some approach such that the extracted social dimensions are sparse.

4.3.1 Communities in Edge-Centric View

It seems reasonable that the number of affiliations one user can participate in is upperbounded by his connections. Consider one extreme case that an actor has only one connection. It is expected that he is probably active in only one affiliation. It is not necessary to assign a non-zero score for each affiliation. Assuming each connection represents one dominant affiliation, we expect the number of affiliations of one actor is no more than his connections. Rather than defining a community as a set of nodes, we redefine it as *a set of edges*. Thus, communities can be identified by partitioning edges of a network into disjoint sets.

For instance, the two communities in the network in Figure 4.1 can be represented by two edge sets in Figure 4.2, where the dashed edges represent one affiliation, and the remaining edges denote another affiliation. One actor is considered associated with one affiliation as long as any of his connections is assigned to that affiliation. Hence, the disjoint edge clusters in Figure 4.2 can be converted into the social dimensions as the last two columns for edge partition in Table 4.1. Actor 1 is involved in both affiliations under this edge partition scheme.

To extract sparse social dimensions, we partition edges rather than nodes into disjoint sets. The edges of those actors involving in multiple affiliations (e.g., actor 1 in the toy network) are likely to be separated into different clusters. Though the partition in edge view is disjoint, the affiliations in the node-centric view can overlap. Each node can engage in multiple affiliations. In addition, the extracted social dimensions following edge partition are *guaranteed to be sparse*. This is because

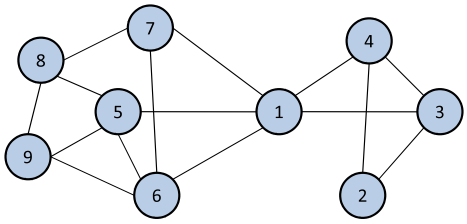


Figure 4.1: A Toy Network

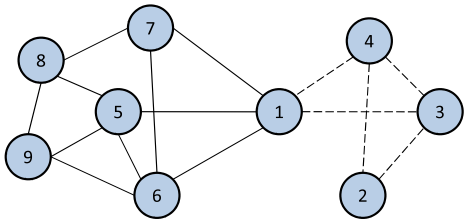


Figure 4.2: Edge Clusters

Table 4.1: Social Dimension(s) of the Toy Example Following Different Approaches

Actors	Modularity Maximization	Edge Partition	
1	-0.1185	1	1
2	-0.4043	1	0
3	-0.4473	1	0
4	-0.4473	1	0
5	0.3093	0	1
6	0.2628	0	1
7	0.1690	0	1
8	0.3241	0	1
9	0.3522	0	1

the number of one's affiliations is no more than her connections. Suppose we have a network with m edges and n nodes, and k social dimensions are extracted. Then each node v_i has no more than $\min(d_i,k)$ non-zero entries in its social dimensions, where d_i is the degree of node v_i. We have the following theorem about the density of extracted social dimensions.

Theorem 4 *Suppose k social dimensions are extracted from a network with m edges and n nodes. The density (proportion of nonzero entries) of the social dimensions based on edge partition is bounded by the following:*

$$density \leq \frac{\sum_{i=1}^{n}\min(d_i,k)}{nk} = \frac{\sum_{\{i|d_i\leq k\}} d_i + \sum_{\{i|d_i>k\}} k}{nk} \tag{4.1}$$

Moreover, for many real-world networks whose node degree follows a power law distribution, the upperbound in Eq. (4.1) can be approximated as follows:

$$\frac{\alpha-1}{\alpha-2}\frac{1}{k} - \left(\frac{\alpha-1}{\alpha-2}-1\right)k^{-\alpha+1} \tag{4.2}$$

where $\alpha > 2$ is the exponent of the power law distribution.

Proof It is observed that the node degree in a social network follows power law [99]. For simplicity, we use a continuous power-law distribution to approximate the node degrees. Suppose

$$p(x) = Cx^{-\alpha}, \quad x \geq 1$$

where x is a variable denoting the node degree and C is a normalization constant. It is not difficult to verify that

$$C = \alpha - 1.$$

Hence,

$$p(x) = (\alpha - 1)x^{-\alpha}, \quad x \geq 1.$$

It follows that the probability that a node with degree larger than k is

$$P(x > k) = \int_k^{+\infty} p(x)dx = k^{-\alpha+1}.$$

Meanwhile, we have

$$\begin{aligned} & \int_1^k xp(x)dx \\ = \; & (\alpha-1)\int_1^k x \cdot x^{-\alpha}dx \\ = \; & \frac{\alpha-1}{-\alpha+2}\, x^{-\alpha+2}|_1^k \\ = \; & \frac{\alpha-1}{\alpha-2}\left(1-k^{-\alpha+2}\right) \end{aligned}$$

Therefore,

$$\begin{aligned} density \; \leq \; & \frac{\sum_{\{i|d_i \leq k\}} d_i + \sum_{\{i|d_i > k\}} k}{nk} \\ = \; & \frac{1}{k}\sum_{d=1}^{k} \frac{|\{i|d_i = d\}|}{n} \cdot d + \frac{|\{i|d_i \geq k\}|}{n} \\ \approx \; & \frac{1}{k}\int_1^k xp(x)dx + P(x > k) \\ = \; & \frac{1}{k}\frac{\alpha-1}{\alpha-2}\left(1-k^{-\alpha+2}\right) + k^{-\alpha+1} \\ = \; & \frac{\alpha-1}{\alpha-2}\frac{1}{k} - \left(\frac{\alpha-1}{\alpha-2} - 1\right)k^{-\alpha+1} \end{aligned}$$

The proof is completed. ∎

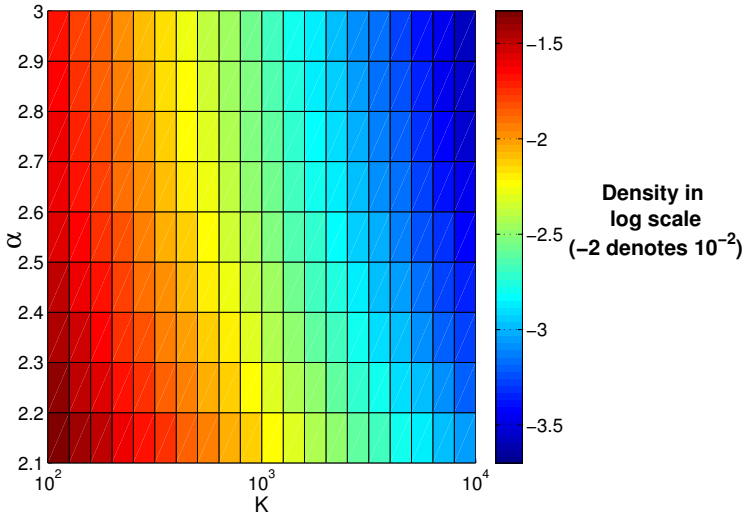


Figure 4.3: Density Upperbound of Social Dimensions

To give a concrete example, we examine a YouTube network[1] with more than 1 million actors and verify the upperbound of the density. The YouTube network has $1,128,499$ nodes and $2,990,443$ edges. Suppose we want to extract $1,000$ dimensions from the network. Since 232 nodes have degree larger than 1000, following Eq.(4.1), the density is upperbounded by

$$(5,472,909 + 232 \times 1,000)/(1,128,499 \times 1,000) = 0.51\%.$$

The node distribution in the network follows a power law with the exponent $\alpha = 2.14$ based on maximum likelihood estimation [99]. Thus, the upperbound following Eq.(4.2) is 0.54%.

Note that the upperbound in Eq. (4.1) is network specific whereas Eq.(4.2) gives an approximate upperbound for a family of networks. It is observed that most power law distributions occurring in nature have $2 \leq \alpha \leq 3$ [99]. Hence, the bound in Eq. (4.2) is valid most of the time. Figure 4.3 shows the function in terms of α and k. Note that when k is huge (close to 10,000), the social dimensions become extremely sparse ($< 10^{-3}$). In reality, the extracted social dimensions is typically even more sparse than this upperbound as shown in later experiments. Therefore, with communities defined in edge view, the extracted social dimensions are sparse, alleviating the memory demand and enabling scalable discriminative learning.

[1]More details are in the experiment part.

Now, the remaining question is how to partition edges efficiently. Below, we will discuss two different approaches to accomplish this task: partitioning a line graph or clustering edge instances.

4.3.2 Edge Partition via Partitioning Line Graph

In order to partition edges into disjoint sets, one way is to look at the "dual" view of a network, i.e., the line graph [54]. We will show that this is not a practical solution. In a line graph L(G), each node corresponds to an edge in the original network G, and edges in a line graph represent the adjacency between two edges in the original graph. Given a network, graph partition algorithms can be applied to its corresponding line graph. The resultant communities in the line graph corresponds to a disjoint edge partition in the original graph. Recently, such a scheme has been used to detect overlapping communities [36, 2]. It is, however, prohibitive to construct a line graph for a mega-scale network. We notice that edges connecting to the same node in the original network form a clique in the corresponding line graph. This property leads to much more edges in a line graph than that in the original network.

Theorem 5 *Let n and m denote the numbers of nodes and connections in a network, and N and M the numbers of nodes and connections in its line graph. It follows that*

$$N = m, \quad M \geq m\left(\frac{2m}{n} - 1\right) \tag{4.3}$$

The equation is achieved if and only if each node in the original graph share the same degree.

Proof Let G and $L(G)$ denote a graph and its line graph respectively, d_i the degree of node i in G. Suppose G has n nodes and m edges, $L(G)$ has N nodes and M edges. As each edge in G becomes a node in $L(G)$, it is straightforward that

$$N = m \tag{4.4}$$

Because the connections of one node in G form a clique in $L(G)$, we have

$$\begin{aligned} M &= \frac{1}{2}\sum_i d_i(d_i-1) \\ &= \frac{1}{2}\sum_i d_i^2 - \frac{1}{2}\sum_i d_i \\ &= \frac{1}{2n}(d_1^2+d_2^2+\cdots+d_n^2)(\underbrace{1+1+\cdots+1}_{n}) - m \\ &\geq \frac{1}{2n}(d_1\cdot 1+d_2\cdot 1+\cdots+d_n\cdot 1)^2 - m \qquad (4.5) \\ &= \frac{1}{2n}4m^2 - m \\ &= m\left(\frac{2m}{n}-1\right) \qquad (4.6) \end{aligned}$$

The inequality (4.5) is derived following Cauchy-Schwarz inequality [55]. The equation is achieved if and only if $d_1 = d_2 = \cdots = d_n$. ∎

For many real-world large-scale networks, the lower bound is never achievable as hardly did we observe any real-world network with the same degree for all nodes. Most networks obey the power law [99]. Consequently, the connections of a line graph tend to multiply without a constant bound with respect to the size of a given network as stated in the following theorem.

Theorem 6 *Let α denote the exponent of a power law distribution for node degrees of a given network, n the size of the network, and M the number of connections in its corresponding line graph. It follows that*

$$E[2M/n]\begin{cases} \quad diverges & if\ 2<\alpha<=3 \\ = \frac{\alpha-1}{(\alpha-3)(\alpha-2)} & if\ \alpha>3 \end{cases} \qquad (4.7)$$

Proof

$$\begin{aligned} E[2M/n] &= E\left[\frac{1}{n}\left(\sum_{i=1}^{n} d_i^2 - \sum_{i=1}^{n} d_i\right)\right] \\ &= E[X^2] - E[X] \end{aligned}$$

where X denotes the random variable (node degree). For a power law distribution $p(x) = (\alpha - 1)x^{-\alpha}$, it follows that [99],

$$E[X] = \frac{\alpha - 1}{\alpha - 2}, \text{if } \alpha > 2;$$
$$E[X^2] = \frac{\alpha - 1}{\alpha - 3}, \text{if } \alpha > 3.$$

Hence,

$$E[2M/n] \begin{cases} \quad diverges & \text{if } 2 < \alpha <= 3 \\ = \frac{\alpha-1}{(\alpha-3)(\alpha-2)} & \text{if } \alpha > 3 \end{cases} \tag{4.8}$$

The divergence of the expectation tells us that as we go to larger and larger data sets, our estimate of number of connections in the line graph with respect to the original network size will increase without bound. ∎

The divergence of the expectation tells us that as we go to larger and larger data sets, our estimate of number of connections with respect to the original network size will increase without bound. As we have mentioned, the majority of large-scale networks follow a power law with α lying between 2 and 3. Consequently, the number of connections in a line graph can be extremely huge. Take the aforementioned YouTube network as an example again. It contains approximate 1 million nodes and 3 million links. Its resultant line graph will contain $4,436,252,282$ connections, too large to be even loaded into memory. Recall that the original motivation to use sparse social dimensions is to address the scalability concern. Now we end up with dealing with a much larger sized line graph. Hence, the line graph is not a suitable representation for practical use. Next, we will present an alternative approach to edge partition, which is much more efficient and scalable.

4.3.3 Edge Partition via Clustering Edge Instances

In order to partition edges into disjoint sets, we treat edges as data instances with their terminal nodes as features. For instance, we can treat each edge in the toy network in Figure 4.1 as one instance, and the nodes that define edges as features. This results in a typical feature-based data format as in Table 4.2. Then a typical clustering algorithm like k-means clustering can be applied to find out disjoint partitions.

Table 4.2: Edge Instances of the Toy Network in Figure 4.1

Edge	Features								
	1	2	3	4	5	6	7	8	9
(1, 3)	1	0	1	0	0	0	0	0	0
(1, 4)	1	0	0	1	0	0	0	0	0
(2, 3)	0	1	1	0	0	0	0	0	0
⋮	.....								

One concern with this scheme is that the total number of edges might be too huge. Owing to the power law distribution of node degrees presented in social networks, the total number of edges is normally linear, rather than square, with respect to the number of nodes in the network. That is, $m = O(n)$ as stated in the following theorem.

Theorem 7 *The total number of edges is usually linear, rather than quadratic, with respect to the number of nodes in the network with a power law distribution. In particular, the expected number of edges is given as*

$$E[m] = \frac{n}{2}\frac{\alpha-1}{\alpha-2}, \tag{4.9}$$

where α is the exponent of the power law distribution.

Proof Suppose a network with n nodes follows a power law distribution as

$$p(x) = Cx^{-\alpha}, \quad x \geq x_{min} > 0$$

where α is the exponent and C is a normalization constant. Then the expected number of degree for each node is [99]:

$$E[x] = \frac{\alpha-1}{\alpha-2}\, x_{min}$$

where x_{min} is the minimum nodal degree in a network. This can be verified via the properties of power law distribution. In reality, we normally deal with nodes with at least one connection, so $x_{min} \geq 1$. Hence, the expected number of connections in a network following a power law distribution is

$$E[m] = \frac{1}{2}\frac{\alpha-1}{\alpha-2}n.$$

∎

Input: data instances $\{x_i | 1 \leq i \leq m\}$
number of clusters k
Output: $\{idx_i\}$

1. construct a mapping from features to instances
2. initialize the centroid of cluster $\{C_j | 1 \leq j \leq k\}$
3. **repeat**
4. Reset $\{MaxSim_i\}$, $\{idx_i\}$
5. **for** j=1:k
6. identify relevant instances S_j to centroid C_j
7. **for** i in S_j
8. compute $sim(i, C_j)$ of instance i and C_j
9. **if** $sim(i, C_j) > MaxSim_i$
10. $MaxSim_i = sim(i, C_j)$
11. $idx_i = j$;
12. **for** i=1:m
13. update centroid C_{idx_i}
14. **until** no change in idx or change of objective $< \varepsilon$

Figure 4.4: Algorithm for Scalable K-means Variant

Still, millions of edges are the norm in a large-scale network. Direct application of some existing k-means implementation cannot handle the problem. E.g., the k-means code provided in Matlab package requires the computation of the similarity matrix between all pairs of data instances, which would exhaust the memory of normal PCs in seconds. Therefore, implementation with an online fashion is preferred.

On the other hand, the data of edge instances is quite sparse and structured. As each edge connects two nodes in the network, the corresponding edge instance has exactly only two non-zero features as shown in Table 4.2. This sparsity can help accelerate the clustering process if exploited wisely. We conjecture that the centroids of k-means should also be feature-sparse. Often, only a small portion of the data instances share features with the centroid. Thus, we only need to compute the similarity of the centroids with their relevant instances. In order to efficiently identify the instances relevant to one centroid, we build a mapping from features (nodes) to instances (edges) beforehand. Once we have the mapping, we can easily identify the relevant instances by checking the non-zero features of the centroid.

By taking care of the two concerns above, we have a k-means variant as in Figure 4.4 for edge-centric clustering. We only keep a vector of *MaxSim* to represent the maximum similarity between

one data instance with a centroid. In each iteration, we first identify the set of relevant instances to a centroid, and then compute similarities of these instances with the centroid. This avoids the iteration over each instance and each centroid, which would cost $O(mk)$ otherwise. Note that the centroid contains one feature (node) if and only if any edge of that node is assigned to the cluster. In effect, most data instances (edge) are associated with few (much less than k) centroids. By taking advantage of the feature-instance mapping, the cluster assignment for all instances (lines 5-11 in Figure 4.4) can be fulfilled in $O(m)$ time. To compute the new centroid (lines 12-13), it costs $O(m)$ time as well. Hence, each iteration costs $O(m)$ time only. Moreover, the algorithm only requires the feature-instance mapping and network data to reside in main memory, which costs $O(m+n)$ space. Thus, as long as the network data can be held in memory, this clustering algorithm is able to partition its edges into disjoint sets. Later as we show, even for a network with millions of actors, this clustering can be finished in tens of minutes while modularity maximization becomes impractical.

As a simple k-means is adopted to extract social dimensions, it is easy to update the social dimensions if the network changes. If a new member joins a network and a new connection emerges, we can simply assign the new edge to the corresponding clusters. The update of centroids with the new arrival of connections is also straightforward. This k-means scheme is especially applicable for dynamic large-scale networks.

4.3.4 Regularization on Communities

The extracted social dimensions are treated as features of nodes. A conventional supervised learning is applied. In order to handle large-scale data with high dimensionality and enormous instances, we adopt linear SVM which can be finished in linear time [37]. Generally, the larger a community size is, the weaker the connections inside the community are. Hence, we would like to build a SVM relying more on communities of smaller sizes. In order to achieve this, we modify typical SVM objective function as follows:

$$\min \lambda \sum_{i=1}^{n} |1 - y_i(\mathbf{x}_i^T \mathbf{w} + b)|_+ + \mathbf{w}^T \Sigma \mathbf{w} \tag{4.10}$$

where λ is a regularization parameter for SVM[2], $|z|_+ = max(0,z)$ represents SVM hinge loss, Σ is a diagonal matrix to regularize the weights assigned to different communities. In particular,

[2]A different notation λ is used to avoid the confusion with community C.

Input: network data, labels of some nodes
Output: labels of unlabeled nodes
1. convert network into edge-centric view as in Table 4.2
2. perform clustering on edges via algorithm in Figure 4.4
3. construct social dimensions based on edge clustering
4. build classifier based on labeled nodes' social dimensions
5. apply regularization on both community size and node affiliation
6. use the classifier to predict the labels of unlabeled ones based on their social dimensions

Figure 4.5: Algorithm for Scalable Learning Based on Sparse Social Dimensions

$\Sigma_{jj} = h(|C_j|)$ is the penalty coefficient associated with community C_j. The penalty function h should be monotonically increasing with respect to community size. In other words, a larger weight assigned to a large community results in a higher cost.

Interestingly, with a little manipulation, the formula in Eq. (4.10) can be solved using standard SVM package with modified input. Let X represent the input data with each row being an instance (e.g., Table 4.1), and $\tilde{\mathbf{w}} = \Sigma^{1/2}\mathbf{w}$. Then the formula can be rewritten as

$$\begin{aligned} & \min \lambda \sum_i |1 - y_i(\mathbf{x}_i^T \mathbf{w} + b)|_+ + \frac{1}{2}\mathbf{w}^T \Sigma^{\frac{1}{2}} \Sigma^{\frac{1}{2}} \mathbf{w} \\ = \; & \min \lambda \sum_i |1 - y_i(\mathbf{x}_i^T \Sigma^{-\frac{1}{2}} \Sigma^{\frac{1}{2}} \mathbf{w} + b)|_+ + \frac{1}{2}\|\tilde{\mathbf{w}}\|_2^2 \\ = \; & \min \lambda \sum_i |1 - y_i(\tilde{\mathbf{x}}_i^T \tilde{\mathbf{w}} + b)|_+ + \frac{1}{2}\|\tilde{\mathbf{w}}\|_2^2 \end{aligned}$$

where $\tilde{\mathbf{x}}_i^T = \mathbf{x}_i^T \Sigma^{-\frac{1}{2}}$. Hence, given an input data matrix X, we only need to left multiply X by $\Sigma^{-\frac{1}{2}}$. It is observed that community sizes of a network tend to follow a power law distribution as shown in the experiment part. Hence, we recommend $h(|C_j|) = \log |C_j|$ or $(\log |C_j|)^2$.

On the other hand, one node is likely to engage in multiple communities. The node affiliation also observes a heavy-tail distribution. Intuitively, a node participating in many communities influences less on other group members compared with those devoted ones. For effective classification learning, we regularize node affiliations by normalizing each nodes social dimensions to sum up to 1. Later in the experiment, we will study which regularization affects more on classification.

In sum, to learn a model for collective behavior, we take the edge-centric view of the network data and partition the edges into disjoint sets. Based on the edge clustering, social dimensions

Table 4.3: Statistics of Social Media Data

Data	BlogCatalog	Flickr	YouTube
Categories	39	195	47
Nodes (n)	10, 312	80, 513	1, 138, 499
Links (m)	333, 983	5, 899, 882	2, 990, 443
Network Density	6.3×10^{-3}	1.8×10^{-3}	4.6×10^{-6}
Maximum Degree	3, 992	5, 706	28, 754
Average Degree	65	146	5

can be constructed and regularization can be applied. Then, discriminative learning and prediction can be accomplished by considering these social dimensions as features. The detailed algorithm is summarized in Figure 4.5.

4.4 Experiment Setup

Two data sets reported in Chapter 2 are used again here. To examine the scalability, we also include a mega-scale network crawled from YouTube [94]. We remove those nodes without connections and select the interest groups subscribed by at least 500 members. Some statistics of the three data sets can be found in Table 4.3. The experiment set up and evaluation measure are the same as in Chapter 2.

As we discussed in Section 4.3.2, constructing a line graph is prohibitive for large-scale networks. Hence, the line-graph approach is not included for comparison. Alternatively, the edge-centric clustering (or $EdgeCluster$) in Section 4.3.2 is used to extract social dimensions on all the data sets. We adopt cosine similarity while performing the clustering. Based on cross validation, the dimensionality is set to 5000, 10000, and 1000 for BlogCatalog, Flickr, and YouTube, respectively. A linear SVM classifier [37] is exploited for discriminative learning.

Another related approach to finding edge partitions is *bi-connected components* [60]. Bi-connected components of a graph are the maximal subsets of vertices such that the removal of a vertex from a particular component will not disconnect the component. Essentially, any two nodes in a bi-connected component are connected by at least two paths. It is highly related to *cut vertices* (a.k.a. *articulation points*) in a graph, whose removal will result in an increase in the number of connected components. Those cut vertices are the bridges connecting different bi-connected components. Thus searching for bi-connected components boils down to searching for articulation

points in the graph which can be solved efficiently by $O(n+m)$. Here n and m represent the number of vertices and edges in a graph respectively. Each bi-connected component is considered a community and converted into one social dimension for learning.

We also compare our proposed sparse social dimension approach with classification performance based on dense representations. In particular, we extract social dimensions according to modularity maximization (denoted as *ModMax*) [121]. *ModMax* has been shown to outperform other representative relational learning methods based on collective inference. We study how the sparsity in social dimensions affects the prediction performance as well as the scalability.

Note that social dimensions allow one actor to be involved in multiple affiliations. As a proof of concept, we also examine the case when each actor is associated with only one affiliation. Essentially, we construct social dimensions based on node partition. A similar idea has been adopted in latent group model [98] for efficient inference. To be fair, we adopt k-means clustering to partition nodes in a network into disjoint sets, and convert the node clustering result as social dimensions. Then, SVM is utilized for discriminative learning. For convenience, we denote this method as *NodeCluster*.

4.5 Experiment Results

In this section, we first examine how the performance varies with social dimensions extracted following different approaches. Then we verify the sparsity of social dimensions and its implication for scalability. We also study how the performance varies with social dimensionality. Finally, concrete examples of extracted social dimensions are given.

4.5.1 Prediction Accuracy

The prediction performance on all the data sets are shown in Table 4.4. The entries in bold face denote the best one in each column. Evidently, *EdgeCluster* is the winner most of the time. Edge-centric clustering shows comparable performance as modularity maximization on BlogCatalog network. yet outperforms *ModMax* on Flickr. *ModMax* on YouTube is not applicable due to the scalability constraint. Clearly, with sparse social dimensions, we are able to achieve comparable performance as that of dense social dimensions. However, the benefit in terms of scalability will be tremendous as discussed in the next subsection.

Table 4.4: Performance on Social Media Networks

BlogCatalog		10%	20%	30%	40%	50%	60%	70%	80%	90%
Micro-F1(%)	*EdgeCluster*	**27.94**	**30.76**	**31.85**	**32.99**	**34.12**	35.00	34.63	35.99	36.29
	BiComponents	16.54	16.59	16.67	16.83	17.21	17.26	17.04	17.76	17.61
	ModMax	27.35	30.74	31.77	32.97	34.09	**36.13**	**36.08**	**37.23**	**38.18**
	NodeCluster	18.29	19.14	20.01	19.80	20.81	20.86	20.53	20.74	20.78
Macro -F1(%)	*EdgeCluster*	16.16	19.16	20.48	**22.00**	**23.00**	**23.64**	23.82	**24.61**	24.92
	BiComponents	2.77	2.80	2.82	3.01	3.13	3.29	3.25	3.16	3.37
	ModMax	**17.36**	**20.00**	**20.80**	**21.85**	22.65	23.41	**23.89**	24.20	**24.97**
	NodeCluster	7.38	7.02	7.27	6.85	7.57	7.27	6.88	7.04	6.83

Flickr		1%	2%	3%	4%	5%	6%	7%	8%	9%
Micro-F1(%)	*EdgeCluster*	**25.75**	**28.53**	**29.14**	**30.31**	**30.85**	**31.53**	**31.75**	**31.76**	**32.19**
	BiComponents	16.45	16.46	16.45	16.49	16.49	16.49	16.49	16.48	16.55
	ModMax	22.75	25.29	27.30	27.60	28.05	29.33	29.43	28.89	29.17
	NodeCluster	22.94	24.09	25.42	26.43	27.53	28.18	28.32	28.58	28.70
Macro-F1(%)	*EdgeCluster*	**10.52**	**14.10**	**15.91**	**16.72**	**18.01**	**18.54**	**19.54**	**20.18**	**20.78**
	BiComponents	0.45	0.46	0.45	0.46	0.46	0.46	0.46	0.46	0.47
	ModMax	10.21	13.37	15.24	15.11	16.14	16.64	17.02	17.10	17.14
	NodeCluster	7.90	9.99	11.42	11.10	12.33	12.29	12.58	13.26	12.79

YouTube		1%	2%	3%	4%	5%	6%	7%	8%	9%
Micro-F1(%)	*EdgeCluster*	**23.90**	**31.68**	**35.53**	**36.76**	**37.81**	**38.63**	**38.94**	**39.46**	**39.92**
	BiComponents	**23.90**	24.51	24.80	25.39	25.20	25.42	25.24	24.44	25.62
	ModMax	—	—	—	—	—	—	—	—	—
	NodeCluster	20.89	24.57	26.91	28.65	29.56	30.72	31.15	31.85	32.29
Macro-F1(%)	*EdgeCluster*	**19.48**	**25.01**	**28.15**	**29.17**	**29.82**	**30.65**	**30.75**	**31.23**	**31.45**
	BiComponents	6.80	7.05	7.19	7.44	7.48	7.58	7.61	7.63	7.76
	ModMax	—	—	—	—	—	—	—	—	—
	NodeCluster	17.91	21.11	22.38	23.91	24.47	25.26	25.50	26.02	26.44

BiComponents, though also separates edges into disjoint sets, which in turn deliver a sparse representation of social dimensions as *EdgeCluster*, yields a poor performance. This is because *BiComponents* outputs highly imbalanced communities. For example, *BiComponents* extracts 271 bi-connected components in the BlogCatalog network. In these 271 components, a dominant one contains $10,042$ nodes while others are all of size 2. When a network is getting more connected (say, Flickr with average degree 146), *BiComponents* is even worse. Only 10 bi-connected components are found. No wonder its Macro-F1 is close to 0. In short, *BiComponents*, though very efficient and scalable, cannot extract informative social dimensions for classification.

The *NodeCluster* scheme, by contrast, forces each actor to be involved in only one affiliation, yielding inferior performance compared with *EdgeCluster*. Among all the compared methods,

EdgeCluster is often the winner. This indicates that its extracted sparse social dimensions do help in collective behavior prediction.

4.5.2 Scalability Study

As we have introduced in Theorem 4, the social dimensions constructed according to edge-centric clustering are guaranteed to be sparse because the density is upperbounded by a small value. Here, we examine how sparse the social dimensions are in practice. We also study how the computational time (with a Core2Duo E8400 CPU and 4GB memory) varies with the number of edge clusters. The computational time, the memory footprint of social dimensions, their density and other related statistics on all the three data sets are reported in Table 4.5.

Concerning the time complexity, it is interesting that computing the top eigenvectors of a modularity matrix actually is quite efficient as long as there is no memory concern. This is observable on the Flickr data. However, when the network scales to millions of nodes (YouTube Data), modularity maximization becomes impossible due to its excessive memory requirement, while our proposed *EdgeCluster* method can still be computed efficiently. The computation time of *EdgeCluster* on YouTube network is much smaller than Flickr, because the YouTube network is extremely sparse and the total number of edges and the average degree in YouTube are actually smaller than those of Flickr as shown in Table 4.3.

Another observation is that the computation time of *Edge-Cluster* does not change much with varying numbers of clusters. No matter what the cluster number is, the computation time of *EdgeCluster* is of the same order. This is due to the efficacy of the proposed k-means variant in Figure 4.4. In the algorithm, we do not iterate over each cluster and each centroid to do the cluster assignment, but exploit the sparsity of edge-centric data to compute only the similarity of a centroid and those relevant instances. This, in effect, makes the computational time independent of the number of edge clusters.

As for the memory footprint reduction, sparse social dimension did an excellent job. On Flickr, with only 500 dimensions, the social dimensions of *ModMax* require 322.1M, whereas *EdgeCluster* requires only less than 100M. This effect is stronger on the mega-scale YouTube network where *ModMax* becomes impractical to compute directly. It is expected that the social

Table 4.5: Scalability Comparison on Social Media Data of Varying Sizes

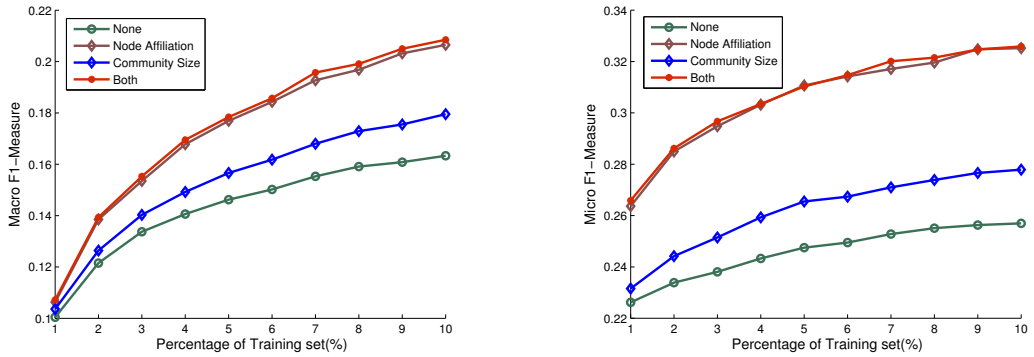
Methods	Time	Space	Density	Upperbound
$ModMax-500$	194.4	41.2M	1	—
$EdgeCluster-100$	300.8	3.8M	1.1×10^{-1}	2.2×10^{-1}
$EdgeCluster-500$	357.8	4.9M	6.0×10^{-2}	1.1×10^{-1}
$EdgeCluster-1000$	307.2	5.2M	3.2×10^{-2}	6.0×10^{-2}
$EdgeCluster-2000$	294.6	5.3M	1.6×10^{-2}	3.1×10^{-2}
$EdgeCluster-5000$	230.3	5.5M	6×10^{-3}	1.3×10^{-2}
$EdgeCluster-10000$	195.6	5.6M	3×10^{-3}	7×10^{-3}

Methods	Time	Space	Density	Upperbound
$ModMax-500$	2.2×10^{3}	322.1M	1	—
$EdgeCluster-200$	1.2×10^{4}	31.0M	1.2×10^{-1}	3.9×10^{-1}
$EdgeCluster-500$	1.3×10^{4}	44.8M	7.0×10^{-2}	2.2×10^{-1}
$EdgeCluster-1000$	1.6×10^{4}	57.3M	4.5×10^{-2}	1.3×10^{-1}
$EdgeCluster-2000$	2.2×10^{4}	70.1M	2.7×10^{-2}	7.2×10^{-2}
$EdgeCluster-5000$	2.6×10^{4}	84.7M	1.3×10^{-2}	2.9×10^{-2}
$EdgeCluster-10000$	1.9×10^{4}	91.4M	7×10^{-3}	1.5×10^{-2}

Methods	Time	Space	Density	Upperbound
$ModMax-500$	N/A	4.6G	1	—
$EdgeCluster-200$	574.7	36.2M	9.9×10^{-3}	2.3×10^{-2}
$EdgeCluster-500$	606.6	39.9M	4.4×10^{-3}	9.7×10^{-3}
$EdgeCluster-1000$	779.2	42.3M	2.3×10^{-3}	5.0×10^{-3}
$EdgeCluster-2000$	558.9	44.2M	1.2×10^{-3}	2.6×10^{-3}
$EdgeCluster-5000$	554.9	45.6M	5.0×10^{-4}	1.0×10^{-3}
$EdgeCluster-10000$	561.2	46.4M	2.5×10^{-4}	5.1×10^{-4}
$EdgeCluster-20000$	507.5	47.0M	1.3×10^{-4}	2.6×10^{-4}
$EdgeCluster-50000$	597.4	48.2M	5.2×10^{-5}	1.1×10^{-4}

dimensions of *ModMax* would occupy 4.6G memory. On the contrary, the sparse social dimensions based on *EdgeCluster* only requires 30-50M.

The steep reduction of memory footprint can be explained by the density of the extracted dimensions. Take YouTube data as an example. When we have 50000 dimensions following edge partition, the density is only 5.2×10^{-5} . Consequently, even if the network has more than 1 million nodes, the extracted social dimensions still occupy tiny memory space. The upperbound of the density is not tight when the number of clusters k is small. As k increases, the bound is getting close to the truth. In general, the true density is roughly half of the estimated bound.



(a) Macro-F1 (b) Micro-F1

Figure 4.6: Regularization Effect on Flickr

4.5.3 Regularization Effect

In this part, we study how *EdgeCluster* performance varies with different regularization schemes. Three different regularizations are implemented: regularization based on community size, node affiliation and both (we first apply community size regularization, then node affiliation). The results without any regularization are also included as a baseline for comparison. In our experiments, the community size regularization penalty function is set to $(\log(|C_i|))^2$, and node affiliation regularization normalizes each node's community membership's summation into 1. As is shown in Figures 4.6, regularization on both community size and node affiliation consistently boost the performance on Flickr data. It is also observed that applying regularization on node affiliation significantly improves performance. It seems like community-size regularization is not as important as the node-affiliation regularization. Similar trends are observed in the other two data sets. After all, we have to point out that, when both community-size regularization and node-affiliation are applied, it is indeed quite similar to the tf-idf weighting scheme for representation of documents [111]. Such a weighting scheme actually can lead to a quite different performance for dealing with social dimensions extracted from a network.

4.5.4 Visualization of Extracted Social Dimensions

As shown in previous subsections, EdgeCluster yields a outstanding performance. But are the extracted social dimensions really sensible? To understand what the extracted social dimensions are, we investigate tag clouds associated with different dimensions. Tag clouds, with font size

(a) *Autos* (b) *Sports*

Figure 4.7: Social Dimensions Selected by *Autos* and *Sports*, Respectively

denoting tags' relative frequency, is widely used in social media websites to summarize the most popular ongoing topics. In particular, we aggregate tags of individuals in one social dimension as the tags of that dimension. However, it is impossible to showcase all the extracted dimensions. Hence, we pick the dimension with the maximum SVM weight given a category.

Due to the space limit, we just show two examples in BlogCatalog. To make the figure legible, we include only those tags whose frequency is greater than 2. The dimension in Figure 4.7a is about *cars*, *autos*, *automobile*, *pontiac*, *ferrari* and etc. It is highly relevant to the category *Autos*. Similarly, the dimension in Figure 4.7b is about *baseball*, *mlb*, *basketball*, and *football*. No wonder it is informative for classification of category *Sports*. There are a few less frequent tags, such as *movies*, and *ipod* associated with selected dimensions as well, suggesting the diverse interests of each person. *EdgeCluster*, by analyzing connections in a network, is able to extract social dimensions that are meaningful for classification.

4.6 Related Work

In order to extract sparse social dimensions for scalable learning, we resort to edge partition. This essentially returns a set of overlapping communities. Finding overlapping communities is attracting increasing attentions. Note that finding overlapping communities is quite a different task from soft clustering [155]. Soft clustering often returns a dense community indicator matrix. It destroys the genuine sparsity presented in a network, thus causing many computational problems. On the contrary, the community indicator matrix of overlapping communities is often quite sparse.

Palla et al. propose a clique percolation method to discover overlapping dense communities [108]. It is composed of two steps: first enumerate all the cliques of size k in a graph; and then find connected cliques. Two k-cliques are connected if they share $k-1$ nodes. Based on the

connections between cliques, we can find the connected components with respect to k-cliques. Each component then corresponds to one community. Since a node can be involved in multiple different k-cliques, the resultant community structure allows one node to be associated with multiple different communities. A similar idea is presented in [113], in which the authors suggest finding out all the maximal cliques in a network, followed by traditional hierarchical clustering.

On the other hand, Gregory [51] extends the Newman-Girvan method [102] to handle overlapping communities. The original Newman-Girvan method recursively removes edges with highest betweenness until a network is separated into prespecified number of disconnected components. But it only outputs non-overlapping communities. Therefore, Gregory proposes to add one more action (node splitting) besides edge removal. The algorithm recursively splits nodes that are likely to reside in multiple communities into two, or removes edges that seem to bridge two different communities. This process is repeated until the network is disconnected into desired number of communities.

The aforementioned methods enumerate all the possible cliques or shortest paths in a network, whose computational cost is daunting for real-world large-scale networks. Recently, a simple scheme proposed to detect overlapping communities is to define communities as a set of edges, instead of nodes [36, 2]. The network can be converted a line graph and then existing methods to find disjoint communities can be applied. However, the scalability of constructing a line graph is prohibitive as we discussed in section 4.3.2.

In our proposed EdgeCluster algorithm, k-means clustering is used to partition the edges of a network into disjoint sets. We also propose a k-means variant to take advantage of its special sparsity structure, which can handle the clustering of millions of edges efficiently. More complicated data structures such as kd-tree [11, 68] can be exploited to accelerate the process. In certain cases, the network might be too huge to reside in memory. Then other k-means variants to handle extremely large data sets like online k-means [4], scalable k-means [19], incremental k-means [106] and distributed k-means [65] can be considered.

4.7 Summary

In this chapter, we examine we can scale up the SocioDim learning framework to predict the online behavior of users in social media, given the behavior information of some actors in the network.

We propose an edge-centric clustering scheme to extract *sparse* social dimensions and a scalable k-means variant to handle edge clustering. Essentially, each edge is treated as one data instance, and the connected nodes are the corresponding features. Then, the proposed k-means clustering algorithm can be applied to partition the edges into disjoint sets, with each set representing one possible affiliation. With this edge-centric view, the extracted social dimensions are warranted to be sparse. Our model based on the sparse social dimensions shows comparable prediction performance as earlier proposed approaches of social dimensions. An incomparable advantage of our model is that it can easily scale to handle networks with millions of actors while the earlier model fails. Recently, an extension to extract sparse social dimensions at multiple resolutions [130] also demonstrates promising result, indicating strong potential of scalable learning based on sparse social dimensions.

Chapter 5

CONCLUSIONS AND FUTURE WORK

Social media is a rich data source of large quantity and high variety. It is a fertile field with many great challenges for data mining. This dissertation proposes the concept of social dimension and presents a social-dimension based learning framework to handle supervised and unsupervised learning in large-scale social media networks. The key contributions of this work are summarized below, followed by future work.

5.1 Key Contributions

Fist, we propose a general supervised learning framework to harness the predictive power of social media networks. Before our work, collective inference was proposed to capture the local dependency of labels between neighboring nodes. However, it treats connections within the network homogeneously. In reality, the connections within a network are often heterogeneous, representing assorted relations. To capture different relations among actors in a network, we propose to extract social dimensions via soft clustering. Based on the extracted social dimensions, a discriminative classifier like SVM can be constructed to determine which dimensions are informative for classification. Extensive experiments on social media data demonstrated that our proposed social-dimension approach (SocioDim) outperforms alternative collective inference, especially when labeled data are few. In the SocioDim framework, a network is converted into attribute format as in conventional data mining. Thus, existing software that is optimized for attribute data can be plugged in to handle social media networks. Moreover, it offers a simple yet effective approach to integrating two types of seemingly orthogonal information: network of actors and actor attributes.

For anther, we extend the general learning framework to handle community detection in social media networks. Based on the concept of social dimensions, we can integrate information in various kinds of social media networks including multi-dimensional networks, multi-modal networks and dynamic networks with evolutions. The unsupervised learning also follows two steps: extract social dimensions from each type of network interaction, modes or snapshots and then integrate them to perform spectral analysis. This unsupervised learning approach connects network anal-

ysis to conventional data mining, thus providing a comprehensible approach to detect groups in various types of social media networks, and leading to a more accurate understanding of human interactions online.

Last but not least, we propose effective solutions to extract sparse social dimensions to make the learning framework scalable to handle real-world large-scale networks. By defining a community in an edge-centric view, we rigorously prove that sparse social dimensions can be obtained via edge partition. EdgeCluster is proposed to take advantage of sparsity in networks. It is extremely scalable to handle sparse large-scale networks, which are common in social media. Such a scalable learning scheme based on sparse social dimensions avoids heavy computation, yet shows comparable performance as that based on dense social dimensions. By sifting through noisy interactions in social media, it provides a viable solution for large-scale social computing.

5.2 Future Work

In the previous chapters, we have discussed about supervised learning, unsupervised learning and scalable learning with social media networks. Following the proposed learning framework, there are many promising directions to explore for future work. We highlight some below.

5.2.1 Effective Extraction of Social Dimensions

The success of our proposed learning framework hinges on social dimensions. Social dimensions can be extracted via soft clustering as discussed in Chapter 2 or EdgeCluster proposed in Chapter 4. Current methods for extracting social dimensions do not use label information. It is not clear whether the supervised extraction of social dimensions would improve the classification performance. Menon and Elkan [93] investigate this question by empirical comparison on several data sets. Quite surprisingly, unsupervised extraction of social dimensions works pretty well, sometimes even better than supervised ones. It requires further investigation to effectively incorporate label information into social dimension extraction.

On the other hand, communities in social media demonstrate strong statistical properties. Kumar et al. [74] found that real-world networks consist of a giant connected components with others being singletons and small-size connected components. Leskovec studied the statistical properties of communities on the giant connected component and found a similar pattern [79]. The optimal

spectral cut always returns a community of size 100 to 200, loosely connected (say, one or two edges) to the remaining of the whole network. What is the implication of these properties to extraction of social dimensions? A further comprehensive comparison of various community detection algorithms is reported in [80]. In these papers, most community detection methods focus on discrete binary cases, i.e., extracting one community from a network based on certain criterion. Whereas SocioDim employs soft clustering to extract social dimensions, and typically many more dimensions instead of just one or two are extracted. We believe a comprehensive comparison of different soft clustering approaches for the extraction of social dimensions is an interesting line of future work.

Furthermore, the current approach requires practitioners to specify the number of social dimensions, and this parameter can be critical to the final classification performance as shown in [122]. Consequently, some procedure like cross-validation has to be used to determine a proper parameter. Considering a network with millions of actors, the community extraction can be time-consuming, as each time a new value is set, the whole community extraction procedure has to be restarted again. It thus remains a challenge to determine the parameter automatically.

In reality, actors are involved in multiple relations of which some are associated with a natural hierarchy. For instance, employees working on a small project form a community, which is within a department, which might reside in another larger community representing the whole company. Similarly, the students of a class form a group, which is within the department group. And the department group resides in another group representing the whole university. These groups at different resolutions can pose assorted regulations on one's behavior [56]. Instead of picking a proper parameter, we conjecture that, by extracting communities at all possible resolutions, we may be able to avoid a tedious cross-validation procedure.

To find communities of varying resolutions, a natural solution is hierarchical clustering. Hierarchies have been used to organize concepts [41] and for classification [131]. However, the overlapping nature of different relations complicates the problem. One user is likely to be involved in multiple different communities, and these communities each reside in a hierarchical path. For example, one student might connect to some of his former classmates online. These classmates span in different departments, which in turn form a university-wide group. At the same time, he might connect to his current colleagues, which also reside in a hierarchical structure. In other words, one

actor is allowed to reside in multiple different paths in the resultant dendrogram, instead of a single one as commonly studied in existing hierarchical community detection approaches. It requires advanced techniques to find *multi-resolution overlapping communities* which lead to sparse social dimensions of different resolutions.

Toward this direction, we have developed a bottom-up hierarchical clustering to extract sparse social dimensions at different resolutions [130]. It considers the local social circle of each user as seed communities and merge them based on community overlap. The resultant classification performance improvement is substantial over that based on predefined number of social dimensions. By exploiting the sparsity present in network connections, the hierarchical clustering can be finished efficiently. Only hours are required to deal with a network of millions of nodes. Essentially, we extract sparse social dimensions at all resolutions. We also hypothesize that by using the label information, we might prune certain branches just like decision trees. At the end, we may obtain a decision tree for network data.

In all, there is no conclusion which method is more effective for the extraction of social dimensions. We expect more research along this direction to emerge in the near future.

5.2.2 Quantifying Crowd Behavior

In Chapter 2, we presented a supervised learning framework that can be utilized to predict the behavior of individuals based on their friends' behavior, such as whether or not one user likes one product, whether or not he supports a presidential candidate, etc. In many applications and domains, however, it is the aggregated mass, in other words, the prevalence of one class, that plays a key role in decision making. Here are some examples:

- In the US presidential election, the candidate who wins a plurality of individual votes in a state wins the state vote.
- A senator might need to collect the mass opinion [156], and prioritize requests based on the urgency and the number of requests [96].
- An accurate estimation of the prevalence of influenza incidents in a certain area can help the authority to allocate attentions, money and resources accordingly [66].

- Companies may want to estimate the proportion of positive or negative responses from customers to take corresponding strategic actions toward a new-generation product.

All the example above share one common characteristic: it is the aggregated mass of certain properties that matters.

The problem of accurately estimating the prevalence of one class in samples is referred as *quantification* [42]. It has been studied in various domains. For example, in the medical statistics field, Zhou et al. estimate the prevalence of a disease in a test population with an imperfect binary diagnostic test with known sensitivity and specificity [161]; Sociologists perform content analysis of blog posts to examine the support with respect to two different candidates [61]; Forman compares various strategies of quantification, aiming at quantifying the counts and costs of different classes in HP customer calls [42] so that HP can allocate human resources accordingly. Quantification is also applied to help semi-supervised learning when labeled samples and unlabeled samples follow different class distributions [153].

Quantification calls for attention because 1) The basic assumption of training and test sharing the same distribution as the foundation in many machine learning techniques are not necessarily true in reality; 2) Many classifiers are optimized for individual predictions. It might induce biases in quantification, especially when the class are imbalanced and insufficient; 3) The accuracy of classifiers can be highly dependent on the availability of training samples, which often involves tremendous human efforts. However, the prevalence might be estimated precisely even with few labeled samples if a robust quantification method is exploited.

All the aforementioned studies focus on quantification when data are presented in conventional attribute format. But in many situations, the data are presented in a relational network, such as the citations among papers, the transportation network and financial transactions between different entities. Recently, with the expanded use of web and social media, oceans of user interaction data are produced in network format. This flood of network data provides valuable opportunities to study collective behavior such as the political views of users, the happiness of people, the opinions and sentiments of online users, the likelihood of product sales online, etc. Essentially, networks offers a new type of information source. With abundant information provided online, we aim to quantify the

collective behavior, i.e., the number of users that are involved in certain type of activities, preferences, or behaviors. Generally, both attribute data (e.g., user tweets, status updates, blog posts, tags, shared content) and network data (e.g., user interaction, friendship network, following/follower network) are available. Ultimately, we hope to exploit both kinds of information collected from social media to quantify the prevalence of users of certain classes.

As an initial attempt, we compared several strategies to estimate prevalence with network data alone in [120]. Two kinds of quantification are presented and compared. 1) Classification-based quantification do not model the prevalence directly, but hinge on post-processing to correct the bias with labels. One specific method Median Sweep (MS) is quite robust and outperforms other methods; 2) The link-based approach relies on link analysis to estimate the class prevalence. It does not require classification and prediction, thus saving tremendous computational cost. But its quantification performance is not comparable to MS. It remains an open problem to model the class prevalence more effectively and efficiently. We hope to encourage more research to address this quantification problem to understand crowd behavior online.

5.2.3 Learning with Streaming Network Data

So far, all the methods we have discussed assume a network is available for learning. In reality, electronic interaction is widespread and frequent. It is likely that a network is growing so rapidly that only streaming methods can handle the learning problem. Ideas of incremental clustering and online learning may be adapted to handle streaming network data.

- Incremental clustering. Incremental spectral clustering [104] can be applied to cluster streaming network data. But this does not change the number of clusters according to incoming data instances. A more appropriate algorithm is COBWEB [41]. Given a sequential presentation of objects and their associated descriptions, COBWEB outputs a hierarchical organization of concepts with a description for each concept. A concept is a collection of objects, corresponding to a cluster. Fisher treats a concept clustering task as a search problem in the hierarchy space. He uses category utility as a heuristic evaluation measure to guide the search, and defines a variety of operators to incorporate objects into a classification tree. Following a hill climbing search strategy, COBWEB explores possible operations when a new object arrives,

and adjusts the concept hierarchy accordingly following the operation which maximizes the utility measure. However, COBWEB is proposed to handle objects with attributes, not network data as studied in this dissertation. It remains a challenge to accomplish unsupervised learning with streaming network interactions.

- Online learning. In our learning framework, social dimensions are extracted and treated as actor attributes for learning. If a network evolves, its social dimensions also changes. Essentially, we have a learning problem in which data instances continuously change its latent feature representation, while in conventional online learning deal with new arriving data instances [117, 22, 75]. Moreover, it is difficult to draw a one-to-one correspondence between latent features between two different timestamps. It is not clear how a classifier constructed based on past network interactions should be updated based on social dimensions extracted from new interaction information.

To conclude, social media offers vast troves of digital information for learning about human beings. It is a young and vibrant field that has shown many promises. However, for the learning tasks presented in this work, only the surfaces are scratched. We expect that more research on learning with large-scale social media networks will emerge to answer challenges from real-world applications.

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Appendix A

COMMUNITY DETECTION

In this chapter, we review existing representative methods for community detection. It is shown that all these methods can be unified with the same process.

Let $G(V,E)$ denote a network with V the set of n vertices and E the m edges, and $A \in \{0,1\}^{n\times n}$ denote the adjacency matrix (network interactions). The degree of node i is d_i. $A_{ij} = 1$ if there is an edge between nodes i and j. Unless specified explicitly, we assume the network is undirected. A community is defined as a group of actors with frequent interactions occurring between them. Community detection attempts to uncover the community membership of each actor. In particular, the problem is defined below:

> **Community Detection:** Given a network $A \in \{0,1\}^{n\times n}$ with n being the number of actors, and k the number of communities in the network, community detection aims to determine the community assignment of each actor. The community assignment is denoted as $H \in \{0,1\}^{n\times k}$ with
>
> $$H_{ij} = \begin{cases} 1, & \text{if actor } i \text{ belongs to community } j \\ 0, & \text{otherwise} \end{cases} \tag{A.1}$$

One common assumption in community detection is that each actor belongs to only one community. That is, $\sum_{j=1}^{k} H_{ij} = 1$. To resolve the community detection problem, various approaches have been developed including latent space models, block model approximation, spectral clustering and modularity maximization. Below, we briefly review these representative methods and show that they can be interpreted in a unified view.

A.1 Latent Space Models

A latent space model maps nodes in a network into a low-dimensional Euclidean space such that the proximity between nodes based on network connectivity are kept in the new space, then the nodes are clustered in the low-dimensional space using methods like k-means [118]. One representative approach is *multi-dimensional scaling* (MDS) [17]. Typically, MDS requires the input of a proximity matrix $P \in \mathbb{R}^{n\times n}$, with each entry P_{ij} denoting the distance between a pair of nodes i and j in the network. For a network, a commonly used proximity measure is *geodesic distance* [147], i.e., the length of the shortest path between two nodes. Let $S \in \mathbb{R}^{n\times \ell}$ denote the coordinates of nodes in the

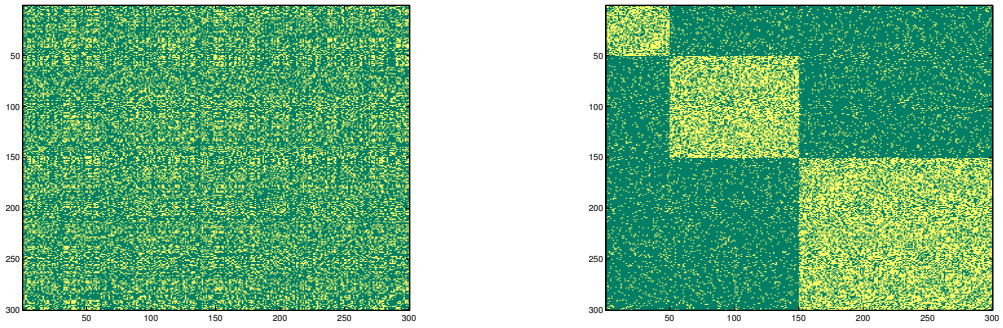

Figure A.1: Basic Idea of Block Model Approximation

ℓ-dimensional space such that S are column orthogonal. It can be shown [17, 110] that

$$SS^T \approx -\frac{1}{2}(I - \frac{1}{n}\mathbf{1}\mathbf{1}^T)(P \circ P)(I - \frac{1}{n}\mathbf{1}\mathbf{1}^T) = \widetilde{P} \tag{A.2}$$

where I is the identity matrix, $\mathbf{1}$ an n-dimensional column vector with each entry being 1, and $\circ$ the element-wise matrix multiplication. It follows that S can be obtained via minimizing the discrepancy between $\widetilde{P}$ and SS^T as follows:

$$\min \|SS^T - \widetilde{P}\|_F^2 \tag{A.3}$$

Suppose V are the top ℓ eigenvectors of $\widetilde{P}$ with largest eigenvalues, Λ a diagonal matrix of top ℓ eigenvalues $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_\ell)$. The optimal S is $S = V\Lambda^{\frac{1}{2}}$. Note that this multi-dimensional scaling corresponds to an eigenvector problem of matrix $\widetilde{P}$. Then classical k-means algorithm can be applied to to S to find community partitions.

A.2 Block Model Approximation

The block model is to approximate a given network by a block structure. The basic idea can be visualized in Figure A.1 where the left graph shows a network and the right one is the block structure after we reorder the index of actors according to their community membership. Each block represents one community. Therefore, we approximate the network interaction A as follows:

$$A \approx S\Sigma S^T \tag{A.4}$$

where $S \in \{0,1\}^{n \times k}$ is the block indicator matrix, Σ the block (group) interaction density, and k the number of blocks. A natural objective is to minimize the following formula:

$$\min \|A - S\Sigma S^T\|_F^2 \tag{A.5}$$

The discreteness of S makes the problem NP-hard. We can relax S to be continuous but satisfy certain orthogonal constraints, i.e., $S^T S = I_k$, then the optimal S corresponds to the top k eigenvectors of A with maximum eigenvalues. Similar to the latent space model, k-means clustering can be applied to S to recover the community partition H.

A.3 Spectral Clustering

Spectral clustering [88] derives from the problem of graph partition. Graph partition aims to find out a partition such that the cut (the total number of edges between two disjoint sets of nodes) is minimized. Though this cut minimization can be solved efficiently, it often returns trivial and non-interesting singletons, i.e., a community consisting of only one node. Therefore, practitioners modify the objective function so that the group size of communities is considered. Two commonly used variants are *Ratio Cut* and *Normalized Cut*. Suppose we partition the nodes of a network into k non-overlapping communities $\pi = (C_1, C_2, \cdots, C_k)$, then

$$\text{Ratio Cut}(\pi) = \sum_{i=1}^{k} \frac{cut(C_i, \bar{C}_i)}{|C_i|} \tag{A.6}$$

$$\text{Normalized Cut}(\pi) = \sum_{i=1}^{k} \frac{cut(C_i, \bar{C}_i)}{vol(C_i)} \tag{A.7}$$

where $\bar{C}_i$ is the complement of C_i, and $vol(C_i) = \sum_{v \in C_i} d_v$. Both objectives attempt to minimize the number of edges between communities, yet avoid the bias of trivial-size communities like singletons. Both can be formulated as a min-trace problem like below

$$\min_{S \in \{0,1\}^{n \times k}} Tr(S^T \widetilde{L} S) \tag{A.8}$$

with $\widetilde{L}$ (graph Laplacian) defined as follows:

$$\widetilde{L} = \begin{cases} D - A & \text{(Ratio Cut)} \\ I - D^{-1/2} A D^{-1/2} & \text{(Normalized Cut)} \end{cases} \tag{A.9}$$

Akin to block model approximation, we solve the following spectral clustering problem based on a relaxation to S.

$$\min_{S} Tr(S^T \widetilde{L} S) \quad s.t.\ S^T S = I_k \tag{A.10}$$

Then, S corresponds to the top eigenvectors of $\widetilde{L}$ with smallest eigenvalues.

A.4 Modularity Maximization

Modularity [101] is proposed specifically to measure the strength of a community partition for real-world networks by taking into account the degree distribution of nodes. Given a random network with n nodes and m edges with desired degree for each node, the expected number of edges between nodes i and j is $d_i d_j/2m$ where d_i and d_j are the degrees of node i and j, respectively. So $A_{ij} - d_i d_j/2m$ measures how far the network interaction between nodes i and j (A_{ij}) deviates from the expected random connections. Given a group of nodes C, the strength of community effect is defined as

$$\sum_{i\in C, j\in C} A_{ij} - d_i d_j/2m.$$

If a network is partitioned into multiple groups, the overall community effect can be summed up as follows:

$$\sum_{C}\sum_{i\in C, j\in C} A_{ij} - d_i d_j/2m.$$

Modularity is defined as

$$Q = \frac{1}{2m}\sum_{C}\sum_{i\in C, j\in C} A_{ij} - d_i d_j/2m. \tag{A.11}$$

where the coefficient $1/2m$ is introduced to normalize the value between -1 and 1. Modularity calibrates the quality of community partitions thus can be used as an objective measure to optimize.

Let $B = A - \frac{\mathbf{d}\mathbf{d}^T}{2m}$, $\mathbf{s}_C \in \{0,1\}^n$ be the community indicator of group C, and S the community indicator matrix, it follows that

$$Q = \frac{1}{2m}\sum_{C}\mathbf{s}_C^T B \mathbf{s}_C = \frac{1}{2m} Tr(S^T B S) = Tr(S^T \tilde{B} S) \tag{A.12}$$

where

$$\tilde{B} = \frac{1}{2m}B = \frac{A}{2m} - \frac{\mathbf{d}\mathbf{d}^T}{(2m)^2}. \tag{A.13}$$

With a spectral relaxation to allow S to be continuous, the optimal S can be computed as the top-k eigenvectors of matrix $\widetilde{B}$ [100] with maximum eigenvalues.

A.5 A Unified View

We briefly present four representative community detection methods: latent space models, block models, spectral clustering and modularity maximization. Interestingly, all these methods can be

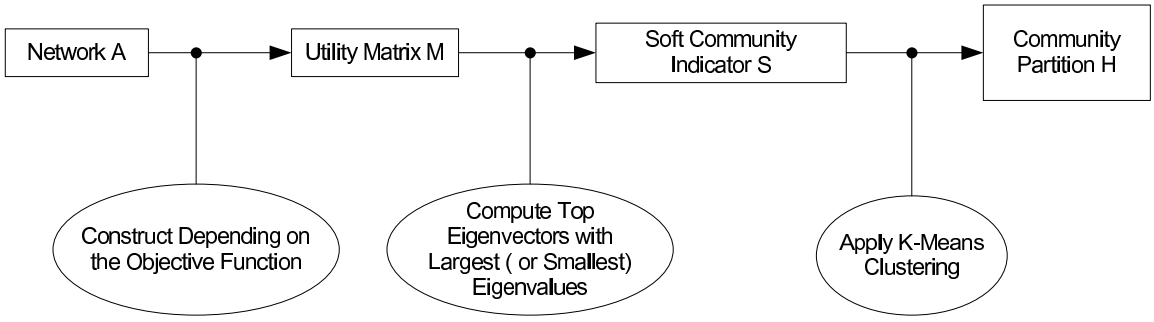

Figure A.2: A Unified View of Representative Community Detection Methods

unified in a process as in Figure A.2. The process is composed of 4 components with 3 intermediate steps. Given a network, a utility matrix is constructed. Depending on the objective function, different utility matrices can be constructed.

$$\text{Utility Matrix } M = \begin{cases} \widetilde{P} \text{ in Eq. (A.2)} & \text{(latent space models)} \\ A \text{ in Eq. (A.4)} & \text{(block model approximation)} \\ \widetilde{L} \text{ in Eq. (A.9)} & \text{(spectral clustering)} \\ \widetilde{B} \text{ in Eq. (A.13)} & \text{(modularity maximization)} \end{cases} \tag{A.14}$$

After obtaining the utility matrix, we obtain the *soft community indicator* S as the top eigenvectors with largest (or smallest subject to formulation) eigenvalues. The selected eigenvectors capture the prominent interaction patterns, representing approximate community partitions. This step can also be considered as a de-noising process since we only keep those top eigenvectors that are indicative of community structures.

A minor issue with the soft community indicator S is its non-uniqueness. Any basis in the top eigenvector space are also a valid solution. Essentially, the solution is equivalent under an orthogonal transformation. Take spectral clustering as an example. Let S be the extracted dimensions based on Eq (A.10), and P be an orthonormal matrix such that $P \in R^{k\times k}, P^TP = PP^T = I_k$. It can be verified that $S' = SP$ is a solution with the same objective:

$$Tr((S')^T\widetilde{L}(S')) = Tr((SP)^T\widetilde{L}(SP)) = Tr(S^T\widetilde{L}SPP^T) = Tr(S^T\widetilde{L}S)$$

This non-uniqueness has to be considered for certain problems.

To recover the discrete partition H, a k-means clustering algorithm is applied. Note that all the aforementioned approaches differ subtly by constructing different utility matrices.

The community detection methods presented above, except the latent space model, are normally applicable to most medium-size networks (say, $100,000$ nodes). The latent space model requires an input of a proximity matrix of the geodesic distance of any pair of nodes, which costs $O(n^3)$ to compute the pairwise shortest path distances. Moreover, the utility matrix of the latent space model is neither sparse nor structured, leading to $O(n^3)$ to compute its eigenvectors. This high computational cost hinders its application to real-world large-scale networks.

On the contrary, the other methods, block model approximation, spectral clustering, and modularity maximization, construct a sparse or structured (a sparse matrix plus low rank update) utility matrix, whose computational cost is almost negligible[1]. Asymptotically, the cost to construct a utility matrix is

$$T_{utility} = O(m). \tag{A.15}$$

Implicitly Restarted Lanczos method (IRLM) can be applied to compute the top eigenvectors efficiently [32, 149]. Let ℓ denote the number of soft community indicators to extract. If one makes the conservative assumption that $O(\ell)$ extra Lanczos steps be involved, IRLM has the worst time complexity of

$$T_{eig} = O(h(m\ell + n\ell^2 + \ell^3)) \tag{A.16}$$

where h, m and n are the number of iterations, the number of edges and nodes in the network, respectively. Typically, $m \sim O(n)$ in a social network with power law distribution [122] and $\ell << n$. In practice, the computation tends to be linear with respect to n if ℓ is small. The post-processing step to extract community partition relies on k-means clustering, which has time complexity

$$T_{kmeans} = O(nk\ell e) \tag{A.17}$$

where ℓ is the number of structural features, e is the number of iterations.

In summary, the representative community detection methods can be unified in the same process. The only difference is how to construct the utility matrix. This also affects the time complexity

[1]The utility matrix of modularity maximization is dense but structured, thus it is rarely computed out. Its structure is exploited directly for eigenvector computation [100, 129].

of different methods. Block model approximation, spectral clustering, and modularity maximization share similar time complexity. They can be solved much more efficiently than latent space models.

Appendix B

CONVERGENCE ANALYSIS

In section 3.5.3, we provide an iterative algorithm to find communities in dynamic multi-mode networks. In this chapter, we study its convergence properties.

Theorem 8 *The objective of formulation* $\mathbf{F_2}$ *in Eq.* (3.22) *following the temporally-regularized multi-mode clustering algorithm in Figure 3.7 is guaranteed to converge.*

Proof Our algorithm essentially implements an alternating optimization strategy for each $C^{(i,t)}$. At each step, we find an optimal solution for one variable while fixing the other variables. Hence, the objective $\mathbf{F_2}$ is non-increasing. Since the objective of $\mathbf{F_2}$ has a lower bound 0, it must converge to a constant value. ∎

However, it is impractical to compute $\mathbf{F_2}$ explicitly. The involved terms $R^t_{i,j} - C^{(i,t)}A^t_{i,j}(C^{(j,t)})^T$ is a full matrix of size $n_i \times n_j$. The temporal regularization term $C^{(i,t)}(C^{(i,t)})^T - C^{(i,t-1)}(C^{(i,t-1)})^T$ is also huge, of size $n_i \times n_i$. It can be problematic to compute them as that might require excessive computational resource. Alternatively, we can compute the equivalent formulation $\mathbf{F_3}$ in (3.26). It involves matrices of size $k_i \times k_j$ or $k_i \times k_i$. Since $k_i << n_i$, $\mathbf{F_3}$ can be computed much more efficiently.

Though the objective converges to a constant, it does not necessarily indicate that the algorithm converges to a (local) optimal, whose gradient is non-negative (non-positive) along with any direction within the feasible domain for a minimization (maximization) problem. For certain problems, the objective function approaches a constant value while the solution is far from even a local optimal (refer to [109] for concrete examples). Below, we further examine the convergence property in terms of community indicators $\{C^{(i,t)}\}$.

It can be shown that C^t_i is guaranteed to converge to a local stationary point under mild conditions. Let $Q^{(i,t)} = C^{(i,t)}(C^{(i,t)})^T$. Then we have some weak convergence result with respect to $\{Q^{(i,t)}\}$. The formulation $\mathbf{F_3}$ in (3.26) can be reformulated in terms of $\{Q^{(i,t)}\}$ as follows:

$$\mathbf{F_4}: \quad \max \quad \sum_{t=1}^{\mathbb{T}} \sum_{1 \le i < j \le m} w_a^{(i,j)} tr\left[Q^{(j,t)} R^T_{i,j} Q^{(i,t)} R_{i,j}\right] + w_b^{(i)} \sum_{t=2}^{\mathbb{T}} \sum_{i=1}^{m} tr\left[Q^{(i,t)} Q^{(i,t-1)}\right] \tag{B.1}$$

$$s.t. \quad Q^{(i,t)} \in \mathscr{M}, \quad i = 1, \cdots, m, \quad t = 1, \cdots, \mathbb{T}$$

$$\mathscr{M}^{(i,t)} = \{S | S = C^{(i,t)}(C^{(i,t)})^T\} \tag{B.2}$$

It can be shown that $Q^{(i,t)}$ converges to a *coordinate-wise optimal point* under some mild conditions. Before we proceed to the details, we would like to introduce some basic concepts and results of *block coordinate descent* (BCD) method [141] (a.k.a. *alternating optimization* [13]). Without loss of generality, we discuss the case of minimizing a function $f(x_1, x_2, \cdots, x_N)$ with each x_i denoting one block of variables. In each iteration, BCD optimizes one block of variables while fixing the other blocks. A basic *cyclic rule* is to choose the same block for iterations k, $k+N$, $k+2N$, $\cdots$, for $k = 1, \cdots, N$. We restate Theorem 4.1(c) in [141] as a lemma below:

Lemma 1 *Assume that the level set $X^0 = \{x : f(x) \leq f(x^0)\}$ is compact and that f is continuous on X^0. Then the sequence $\{x^r = (x_1^r, x_2^r, \cdots, x_N^r)\}_{r=0,1,\cdots}$ generated by block coordinate descent is defined and bounded. Moreover, if $f(x_1, \cdots, x_N)$ has at most one minimum in x_k for $k = 2, \cdots, N-1$, and the cyclic rule is used, then every cluster point z of $\{x^r\}_{r \equiv (N-1) \bmod N}$ is a coordinate-wise minimum point of f.*

Based on the lemma, we have the following theorem:

Theorem 9 *$Q^{(i,t)}$ converges to a coordinate-wise optimal point following our algorithm, if the k_i-th and (k_i+1)-th singular values of P_i^t defined in Eq. (3.27) are different in each iteration.*

Proof Clearly, $\mathbf{F_4}$ in (B.1) is continuous with respect to $Q^{(i,t)}$. If the k_i and (k_i+1)-th singular values of P_i^t defined in Eq. (3.27) are different, we have a *unique* $Q^{(i,t)}$ that equals to the outer product of the top k_i^t left singular vectors of P_i^t. If this is always the case, i.e., $\mathbf{F_4}$ has at most one minimum for each block optimization, $Q^{(i,t)}$ converges to a coordinate-wise minimum point of $\mathbf{F_4}$ according to Lemma 1. ∎

Theorem 10 *Our temporally-regularized clustering algorithm approaches a stationary point for $\{C^{(i,t)}\}$ if the k_i-th and (k_i+1)-th singular values of P_i^t defined in Eq. (3.27) are different in each iteration.*

Proof Based on Theorem 9, $\{Q^{(i,t)}\}$ converges to a coordinate-wise minimum point if the k_i-th and (k_i+1)-th singular values of P_i^t are always different. Let $C^{(i,t)}$ be the corresponding decomposition of the limiting point $Q^{(i,t)}$ following Eq. (B.2). To prove that the limiting point $C^{(i,t)}$ is a stationary

point, we can show that the gradient of the objective function within the feasible domain is 0. Specifically, we show that the gradient of the objective function projected into the tangent space of the feasible domain is 0. For notation convenience, we write $C^{(i,t)}$ as C, and M_i^t as M in the subsequent proof.

Let $Q^{(i,t)}$ denote the limiting point, i.e. a coordinate-wise optimal point. Its corresponding C must satisfy Theorem 3. In other words,

$$MC = C\Lambda \tag{B.3}$$

where Λ denotes a diagonal matrix of the top eigenvalues of M. Following Eq. (3.28), $\mathbf{F_3}$ in terms of C can be written as

$$\mathbf{F_3}(C) = tr\left[C^T MC\right] + const$$

with M defined in Eq. (3.30). So the gradient of $\mathbf{F_3}$ with respect to C is

$$g(C) = 2MC. \tag{B.4}$$

Recall that $C \in \mathbb{R}^{n_i \times k_i}$ is column orthogonal, which forms a smooth Stiefel manifold [35] of dimension $n_i k_i - k_i(k_i+1)/2$. The projection of any $Z \in \mathbb{R}^{n_i \times k_i}$ onto the tangent space of the manifold [29] is:

$$\pi_{\mathscr{T}}(Z) := C\frac{C^T Z - Z^T C}{2} + (I - CC^T)Z. \tag{B.5}$$

Hence, the gradient $g(C)$ projected into C's tangent space is

$$\begin{aligned}
& \pi_{\mathscr{T}}(g(C)) \\
= \;& C\frac{2C^T MC - 2C^T M^T C}{2} - 2(I - CC^T)MC \\
= \;& -2(I - CC^T)MC && \text{(B.6)} \\
= \;& -2(I - CC^T)C\Lambda && \text{(B.7)} \\
= \;& 0
\end{aligned}$$

where Eq. (B.6) follows from the symmetry of M, and Eq. (B.7) follows from the fact that C corresponds to the top eigenvectors of M as stated in Eq. (B.3). Hence, the projected gradient of $\mathbf{F_3}$ with

respect to C within the feasible domain (the Stiefel manifold) is zero. Since the objective function is continuously differentiable and the projected gradient is zero for all the blocks of variables, the final solution of $\{C\}$ is a stationary point (critical point). ∎

In Theorems 9 and 10, we require each block optimization step to satisfy certain properties for singular values. A simple strategy to check whether or not our solution is a stationary point, is to verify whether there is any case that the k_i and (k_i+1)-th singular values of P_i^t are exactly the same. In real-world multi-mode networks, this rarely happens due to noise. Hence, this convergence result is valid in general.

We want to emphasize that it is improper to use the difference of $C^{(i,t)}$ between iterations as a guide for convergence verification. As shown in Theorem 3, the optimal C_i^t corresponds to the top left-singular vectors, which is *not unique*. But this does not refrain the algorithm from reaching a stationary point at the end. A proper convergence criterion is the difference of $Q^{(i,t)}$ between iterations is sufficiently small. But $Q^{(i,t)}$ is a full matrix of size $n_i \times n_i$, which might be computationally expensive. Since the obtained $C^{(i,t)}$ is an approximate community indicator matrix, exact convergence may not be necessary. For practical use, it works fine to simply check whether the objective value of $\mathbf{F}_3$ stabilizes.